Benchmarking proximal methods for CS-acquired MR image analysis reconstruction

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1. Introduction

Magnetic resonance imaging (MRI) is one of the most powerful imaging techniques for examining the human body since it allows early and accurate diagnosis of pathologies. Although high magnetic field systems ($\geq$ 3 Tesla) enable increased spatial resolution, long scan times and motion sensitivity continue to impede the exploitation of HR-MRI. To circumvent that problem, Compressed Sensing (CS) was introduced [5], among other techniques, to reduce the acquisition time, taking advantage of the structure of MR images. However, the time gained on acquisition has been lost on reconstruction as sparse recovery amounts to iteratively solving a linear inverse problem. The motivation to reduce the time required for image reconstruction is the following: the MRI physician needs to analyze the reconstructed image to know if there has been some movement causing some motion artifacts, and therefore take the decision to rerun the exam. Thus, decreasing reconstruction time would mean reducing the overall duration of the MRI exam. This would also help approaching the goal of real-time MRI, which is useful for monitoring cardiovascular procedures for example. This becomes substantially important when going from 2D to 3D to 4D (3D + time or contrast) with very high-resolution, namely 1024x1024 or 250 µm in plane resolution at ultra-high fields ($\geq$ 7 Tesla). The recent development [4] of acceleration techniques for FISTA, an algorithm to solve the reconstruction problem, could help improve the reconstruction speed.

2. The reconstruction problem

$$\underset{\boldsymbol{x} \in \mathbb{C}^{n \times n}}{\arg\min}\, \mathcal{J}(\boldsymbol{x}) = \frac{1}{2}\|\boldsymbol{y} - \boldsymbol{F}_\Omega \boldsymbol{x}\|_2^2 + \lambda \|\boldsymbol{\Psi x}\|_1 \quad (1)$$

where $\boldsymbol{x}$ denotes the sought complex-valued MR image, n is its dimension, F_Ω is the Fourier operator, possibly non-uniform and under-sampled over the non-Cartesian set Ω, $\boldsymbol{y}$ the MR Fourier data also called k-space samples. λ refers to the regularization parameter and $\boldsymbol{\Psi}$ to the wavelet decomposition operator as the MR image is assumed to be sparse (at least compressible) in the wavelet basis [5].

For ease of notation, we denote: $F(\boldsymbol{x}) = \frac{1}{2}\|\boldsymbol{y} - \boldsymbol{F}_\Omega \boldsymbol{x}\|_2^2$ and $R(\boldsymbol{x}) = \lambda\|\boldsymbol{\Psi x}\|_1$

3. Faster FISTA [4]

Algorithm: FISTA variations to solve Eq. (1) (contributions of [4] highlighted in red)

Input: $greedy \in \{1; 0\}$; $\gamma_k \in]0, \frac{1+greedy}{L}]$; $p, q \in]0,1]$; $r \in]0,4]$; $\xi \in]0,1[$; $S > 1$

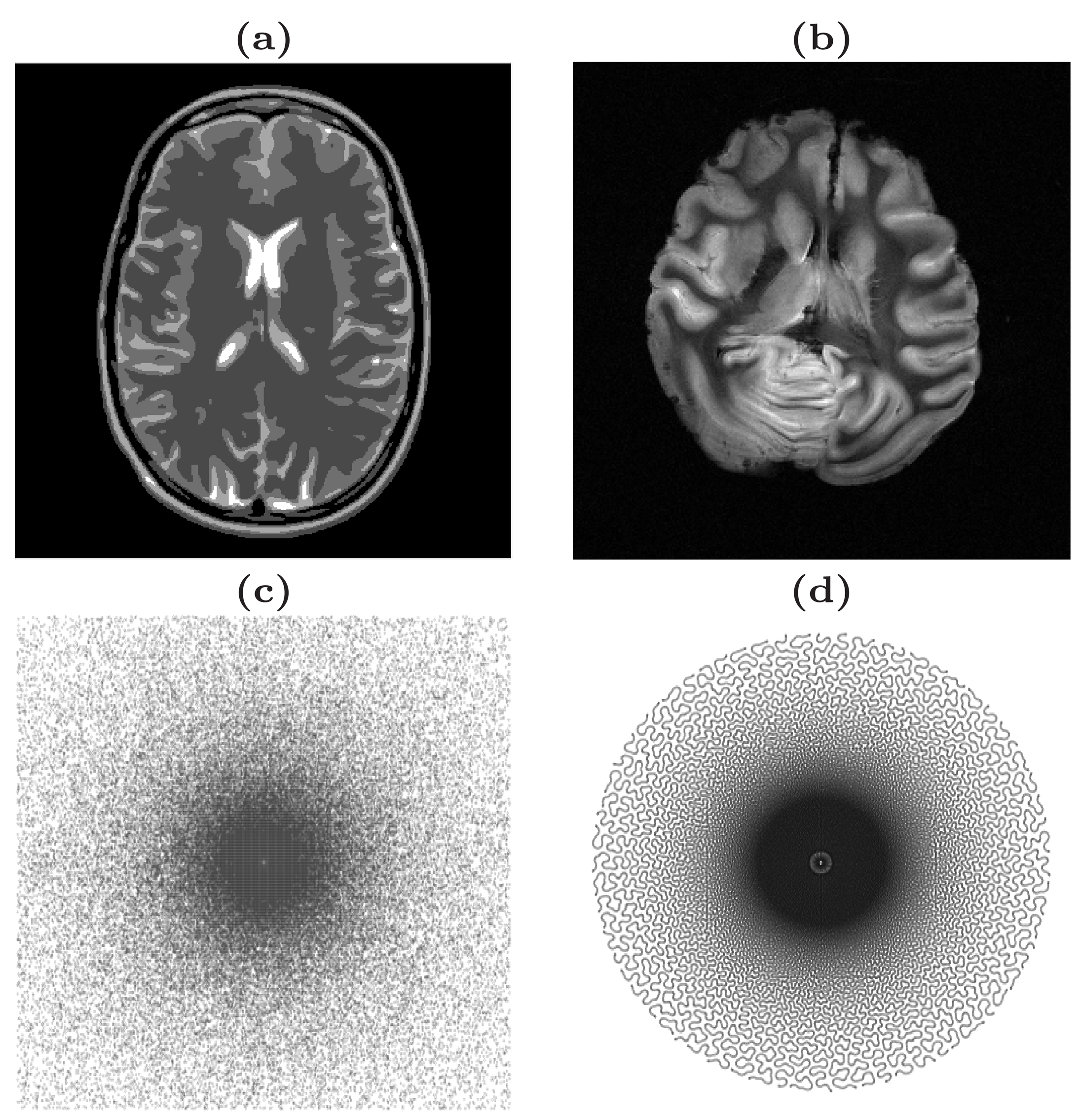
$t_0 = 1$

while *Not converged* **do**

- $t_k = \frac{p + \sqrt{q + r t_{k-1}^2}}{2}$; $a_k = \frac{t_{k-1} - 1}{t_k}$
- **if** $greedy = 1$ **then**
  - $a_k = 1$
- **end**
- $z_k = x_k + a_k(x_k - x_{k-1})$; $x_{k+1} = \mathrm{prox}_{\gamma_k R}(z_k - \gamma_k \nabla F(z_k))$
- **if** $(z_k - x_{k+1})^T (x_{k+1} - x_k) \geq 0$ **then**
  - $r = \xi r$; $z_k = x_k$
- **end**
- **if** $greedy = 1$ *and* $\|x_{k+1} - x_k\|_2 \geq S\|x_1 - x_0\|_2$ **then**
  - $\gamma_{k+1} = \max\{\xi \gamma_k; \frac{1}{L}\}$
- **end**

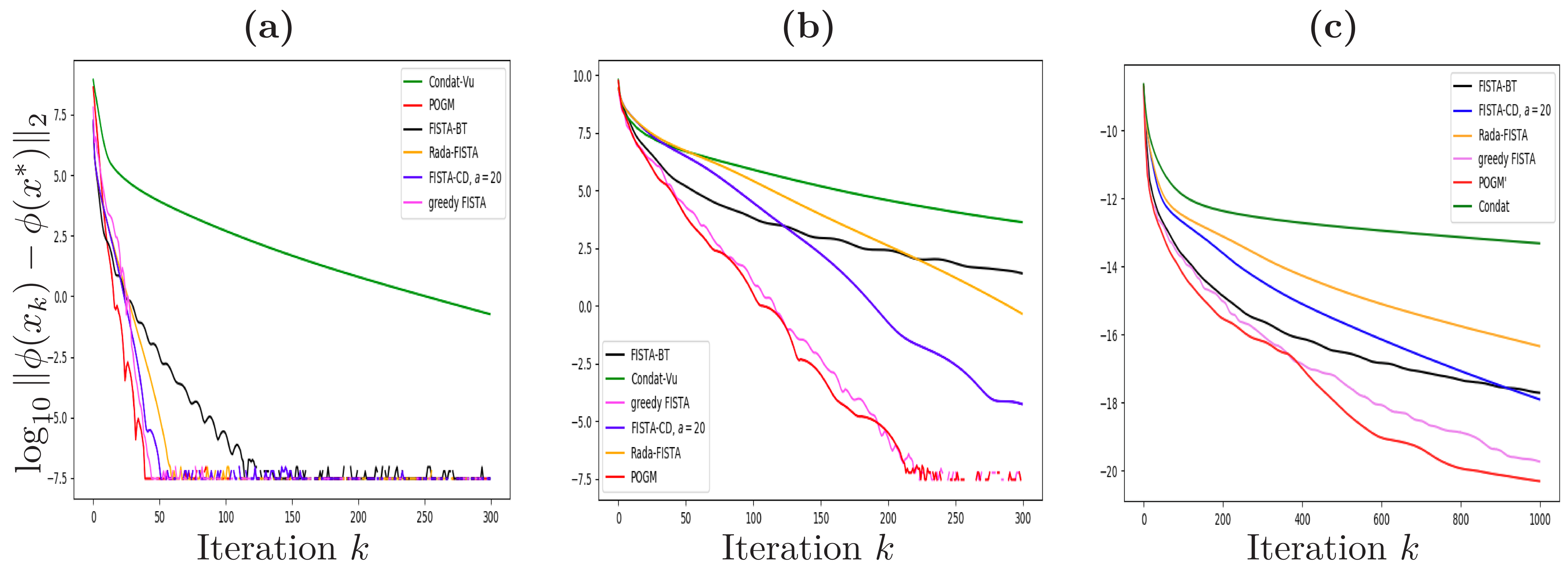
end

4. Data



(a) 2D MRI phantom (512×512)
(b) ex-vivo Cartesian baboon brain T_2^*-weighted at 7T
(c) Variable Density (VD) sampling scheme (25% under-sampling), non-realistic
(d) SPARKLING [3] sampling scheme (Acceleration Factor of 15)

5. Results



Comparison of the convergence speed of different algorithms to solve Eq. 1. Parameters used for: 1) FISTA-CD (by Chambolle and Dossal): $a = 20$; 2) Rada-FISTA: $p = \frac{1}{30}$, $q = \frac{1}{10}$, $\xi = 0.96$; 3) greedy FISTA: $\xi = 0.96$, $S = 1.1$; 4) POGM' [2]: $\overline{\sigma} = 0.96$. 5) Condat-Vu [1]: $\rho = 1$, $\sigma = 10$. The reference is the original implementation proposed by Beck and Teboulle (FISTA-BT). Greedy-FISTA and POGM' converge faster.

(a) Results for the 2D MRI phantom with the VD sampling
(b) Results for the 2D MRI phantom with the SPARKLING sampling
(c) Results for the baboon brain with a retrospective SPARKLING undersampling

6. Conclusions

The results presented show that the greedy FISTA compares to POGM' in terms of iterations (only a few more needed for greedy FISTA to converge). The complexity per iteration is slightly higher for greedy FISTA since it needs to compute an additional norm compared to POGM'. However, greedy FISTA theoretically uses twice less memory. This might be of a huge importance when scaling these algorithms to 3D parallel imaging. In the latter context, 32 to 64 k-space are collected simultaneously over multiple receivers, each of them going up to 512^3 in dimension. The results also confirm the interest of using greedy FISTA compared to vanilla FISTA.

7. References

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