OSCAR-based Reconstruction for Compressed Sensing and Parallel MR Imaging

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ISMRM 2019, Montreal, CAN
Outline

1. Motivation & Context
   - Why non-Cartesian acquisition
   - Non-Cartesian MR image reconstruction in parallel imaging

2. Calibration-less MR image reconstruction
   - Problem statement
   - Joint sparsity regularization

3. Experiments & Results
   - Experimental set-up
   - Results

4. Conclusion & Outlook
Anatomical MRI is generally acquired using Cartesian sampling.

Figure: Typical (a) Cartesian (b) parallel acquisition (c) CAIPIRINHA\(^1\) acquisition

... however in some cases non-Cartesian trajectories are useful ...

\(^1\)Breuer et al. 2006, *Magnetic Resonance in Medicine.*
Non-Cartesian trajectories for anatomical MRI

A non-exhaustive list of usages

- For ultra-short echo time imaging\(^2\)
- X-nuclei imaging (TPI\(^3\))
- To correct for motion, especially for abdominopelvic MRI\(^4\)

![Figure: Stack of stars used for VIBE acquisition](image-url)

Non-Cartesian trajectories for anatomical MRI

A non-exhaustive list of usages

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Figure: A: Free-breathing stack-of-stars VIBE, B: Breath-holding conventional VIBE

Non-Cartesian trajectories for anatomical MRI

Renewed interest to speed-up acquisition in the context of Compressed Sensing\(^5\).

\textbf{Figure:} Example of non-Cartesian trajectories.

\(^5\)Lazarus et al. 2019, \textit{Magnetic Resonance in Medicine}. 
Non-Cartesian trajectories for anatomical MRI

Renewed interest to speed-up acquisition in the context of Compressed Sensing\textsuperscript{5}.

\textbf{Figure}: Comparison of different acquisition trajectories with 16-fold accelerated acquisition on T2*-weighted images.

\textsuperscript{5}Lazarus et al. 2019, \textit{Magnetic Resonance in Medicine}.
Parallel imaging acquisition: collect multiple k-space data using a multi-receiver coil as the latter is known to boost the SNR.

Illustration of multi-receiver coil (phased array).

How do we reconstruct MR images from non-Cartesian k-space measurements in parallel imaging?
Non-Cartesian reconstruction techniques can be split in two categories:

1. Self-calibrating methods:

   \[
   \hat{x} = \arg \min_{x \in \mathbb{C}^{N \times N}} \sum_{\ell=1}^{L} \sigma_{\ell}^{-2} \| F_{\Omega} S_{\ell} x - y_{\ell} \|_2^2 + \lambda \| \Psi x \|_1
   \]

   \( y_{\ell} \in \mathbb{C}^{M} \) the \( \ell \)th channel-specific data set

   \( x \in \mathbb{C}^{N \times N} \) the reconstructed image (e.g. \( N = 512 \times 512 \))

   \( F_{\Omega} \) the forward under-sampling Fourier operator

   \( \Psi \in \mathbb{C}^{N \times N} \) linear operator related to a sparse decomposition

   Note: Extraction of coil sensitivity maps is challenging in non-Cartesian case

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Non-Cartesian MR image reconstruction in parallel imaging

Self-calibrating methods

Non-Cartesian reconstruction techniques can be split in two categories:

1. Self-calibrating methods:
   - require a region where the signal has been sampled at least at the Nyquist rate

\[ \hat{x} = \arg\min_{x \in \mathbb{C}^{N \times N}} \sum_{\ell=1}^{L} \sigma - \frac{1}{2} \| F_{\Omega} S_{\ell} x - y_{\ell} \|_2^2 + \lambda \| \Psi x \|_1 \]

- \( y_{\ell} \in \mathbb{C}^{M} \) the \( \ell \)th channel-specific data set
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Non-Cartesian MR image reconstruction in parallel imaging

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   - model the coil sensitivity profiles $S_\ell$ for all channels $\ell = 1, \ldots, L$\(^7\),\(^8\)
   - solve an inverse problem and recover a single full FOV image:

   \[
   \hat{x} = \arg \min_{x \in \mathbb{C}^N} \frac{1}{2} \sum_{\ell=1}^{L} \sigma_\ell^{-2} \| F_\Omega S_\ell x - y_\ell \|^2_2 + \lambda \| \Psi x \|_1
   \]  

   \(y_\ell \in \mathbb{C}^M\) the $\ell^{th}$ channel-specific data set
   \(x \in \mathbb{C}^N\) the reconstructed image (ex. $N = 512 \times 512$)
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Non-Cartesian MR image reconstruction in parallel imaging

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Non-Cartesian reconstruction techniques can be split in two categories:

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  - do not require any calibration region

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  - solve an inverse problem but recover channel-specific images

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Non-Cartesian MR Image reconstruction from multi-channel array coil acquisition

Calibration-less methods

Non-Cartesian reconstruction techniques can be split in two categories:

2. Calibration-less methods:
   - do not require any calibration region
   - solve an inverse problem but recover channel-specific images
   - use the redundant information given by each coil to impose constraints such as low-rank CLEAR\(^9\) or group-sparsity CALM\(^10\)

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\(^{10}\) Majumdar and Ward 2012, *Magnetic Resonance in Medicine*.
Non-Cartesian MR Image reconstruction from multi-channel array coil acquisition

Calibration-less methods

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- Calibration-less methods:
  - do not require any calibration region
  - solve an inverse problem but recover channel-specific images
  - use the redundant information given by each coil to impose constraints such as low-rank CLEAR$^9$ or group-sparsity CALM$^{10}$
  - more likely to be used for on-line image reconstruction

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Calibration-less MR image reconstruction problem solved using an *analysis formulation*:

**Definition**

MR image reconstruction is formulated as follows:

\[
\hat{x} = \arg\min_{x \in \mathbb{C}^{N \times L}} \left\{ \frac{1}{2} \sum_{\ell=1}^{L} \sigma^{-2}_{\ell} \| F_{\Omega} x_{\ell} - y_{\ell} \|_{2}^{2} + g(Tx) \right\},
\]

(2)

with:

- \( y_{\ell} \in \mathbb{C}^{M} \) the \( \ell \)\(^{th} \) channel-specific data set
- \( x_{\ell} \in \mathbb{C}^{N} \) the \( \ell \)\(^{th} \) channel-specific reconstructed image (ex. \( N = 512 \times 512 \))
- \( F_{\Omega} \) is the forward under-sampling Fourier operator
- \( T \in \mathbb{C}^{N_{\Psi} \times N} \) linear operator related to a sparse decomposition
- \( g \) is a convex regularization term that promotes sparsity
We aim to find:

$$\hat{x} \in \arg\min_{x \in \mathbb{C}^{N \times L}} [f(x) + g(Tx)]$$

(3)

where:

- $f$ is convex, differentiable on $\mathbb{C}^{N \times L}$ and its gradient is $\beta$-Lipschitz
- $g \in \Gamma_0(\mathbb{C}^{N \Psi \times L})^{11}$ with a closed form proximity operator, given by:

$$\text{prox}_g(z) = \arg\min_{v \in \mathbb{C}^{\Psi \times L}} \frac{1}{2} \|z - v\|^2 + g(v)$$

(4)

Note: Those are standard assumptions in optimization based image reconstruction methods.

\[ \text{11} \Gamma_0 \text{ is the set of convex proper lower semi-continuous functions on } \mathbb{C}^{\Psi \times L} \text{ taking values on } \mathbb{R} \cup \text{inf} \]
Optimization algorithm

Condat-Vũ sequence

Using a primal-dual optimization method proposed by Condat-Vũ\cite{condat2013, vu2013}:

\textbf{Algorithm 1: Condat-Vũ algorithm}

\begin{algorithm}
\begin{align*}
\text{initialize} & \quad k = 0, \tau > 0, \kappa > 0, \mathbf{x}_0, \mathbf{z}_0; \\
\text{while} & \quad k \leq K \text{ do} \\
& \quad \mathbf{x}_{k+1} := \mathbf{x}_k - \tau (\nabla f(\mathbf{x}_k) + \mathbf{T}^* \mathbf{z}_k); \\
& \quad \mathbf{w}_{k+1} := \mathbf{z}_k + \kappa \mathbf{T} (2\mathbf{x}_{k+1} - \mathbf{x}_k); \\
& \quad \mathbf{z}_{k+1} := \mathbf{w}_{k+1} - \kappa \text{prox}_g/\kappa \left( \frac{\mathbf{w}_{k+1}}{\kappa} \right); \\
\text{end}
\end{align*}
\end{algorithm}

with:

- the algorithm weakly converges to the solution of Eq. (3) if

\[
\frac{1}{\tau} - \kappa \|\mathbf{T}\|^2 \geq \frac{\beta}{2}
\]

- \(\tau\) and \(\kappa\) hyper-parameters set as follows: \(\tau := \frac{1}{\beta}\), \(\kappa := \frac{\beta}{2\|\mathbf{T}\|^2}\)

\cite{condat2013, vu2013} Condat 2013, \textit{Journal of Optimization Theory and Applications}.

\cite{vu2013} Vũ 2013, \textit{Advances in Computational Mathematics}.  

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Joint sparsity regularization
Group-LASSO

Parallel imaging has been proved to have tighter recovery guarantees than single channel acquisition when combined with Group-LASSO (GL) regularization\textsuperscript{14}.

\textbf{Definition}

The group-LASSO penalty is defined as follows:

\[
g_{GL}(z) = \|z\|_{1,2} = \sum_{s=1}^{S} \left( \lambda \gamma^s \sum_{p=1}^{P_s} \sqrt{\sum_{\ell=1}^{L} |z_{sp\ell}|^2} \right)
\]

- $\lambda$ and $\gamma$ are positive hyper-parameters
- $s$ models the scale or subband dependence

Joint sparsity regularization
Sparse group-LASSO

Variant: Sparse group-LASSO\textsuperscript{16} (sGL)

Definition

\[ \forall \tilde{z} \in \mathbb{C}^{N_x \times L}, g_{sGL}(\tilde{z}) = g_{GL}(\tilde{z}) + \mu \| \tilde{z} \|_1 \] (5)

\[ \mu \] being positive hyper-parameter.

sGL proximity operator\textsuperscript{18} is closed form and corresponds to the composition of GL proximity operator with soft-thresholding.

Joint sparsity regularization
Octagonal Shrinkage and Clustering Algorithm for Regression

Inferring the structure via a pairwise $\ell_\infty$ norm. OSCAR regularization\(^\text{17}\) is defined as follows:

\[
g_{\text{OSCAR}}(z) = \sum_{s=1}^{S} \lambda \left[ \sum_{j=1}^{P_sL} |z_{sj}| + \gamma \sum_{j<k} \max\{|z_{sj}|, |z_{sk}|\} \right]
\]

\[
= \sum_{s=1}^{S} \lambda \left[ \sum_{j=1}^{P_sL} (\gamma(j - 1) + 1) |z_{sj}| \right] \tag{6}
\]

where:
- $z_{\downarrow} \in \mathbb{C}^{N_\psi \times L}$ the wavelet coefficients sorted in decreasing order, i.e.: $\forall s \in \mathbb{N}, |z_{s1}| \leq \cdots \leq |z_{sP_sL}|$.
- $\lambda$ and $\gamma$ are some positive hyper-parameters that need to be set.

\(^\text{17}\)Bondell and Reich 2008, *Biometrics*. 

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\begin{align}
g_{\text{OSCAR}}(z) &= \sum_{s=1}^{S} \lambda \left[ \sum_{j=1}^{P_s L} |z_{sj}| + \gamma \sum_{j<k} \max\{ |z_{sj}|, |z_{sk}| \} \right] \\
&= \sum_{s=1}^{S} \lambda \left[ \sum_{j=1}^{P_s L} (\gamma(j - 1) + 1) |z_{sj}|_\downarrow \right] \tag{6}
\end{align}

where:
- $z_\downarrow \in \mathbb{C}^{N_\psi \times L}$ the wavelet coefficients sorted in decreasing order, i.e.: $\forall s \in \mathbb{N}, |z_{s1}| \leq \cdots \leq |z_{sP_s L}|$.
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\(^{17}\)Bondell and Reich 2008, *Biometrics.*
Experimental set-up

Sequence parameters:

- Ex-vivo baboon brain
- 7T Siemens Scanner GRE
- 1Tx/32Rx Nova coil
- Sparkling trajectory
- 390\(\mu\)m \(\times\) 390\(\mu\)m in plane-resolution
- 3mm slice thickness
- Acceleration factor of 15 in time
- Under-sampling factor of 2.5
- \(T\): Undecimated Bi-Orthogonal 7-9 wavelet transform

Hyper-parameters set using a grid-search procedure. Cartesian scan 512\(\times\)512 was acquired and used for reference.
Coil combination: Square root of the Sum-Of-Squares Structural SIMilarity Index (SSIM)\textsuperscript{18} used to set hyper-parameters

Table: Image quality assessment for all regularizers.

<table>
<thead>
<tr>
<th></th>
<th>SSIM</th>
<th>pSNR (dB)</th>
<th>NRMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>IFT</td>
<td>0.847</td>
<td>26.50</td>
<td>0.263</td>
</tr>
<tr>
<td>GL</td>
<td>0.864</td>
<td>26.92</td>
<td>0.254</td>
</tr>
<tr>
<td>sGL</td>
<td>0.851</td>
<td>26.77</td>
<td>0.259</td>
</tr>
<tr>
<td>OSCAR</td>
<td>0.875</td>
<td>30.49</td>
<td>0.177</td>
</tr>
<tr>
<td>$\ell_1$-ESPIRiT</td>
<td>0.874</td>
<td>28.32</td>
<td>0.238</td>
</tr>
</tbody>
</table>

Note: $\ell_1$-ESPIRiT is a self-calibrating method.

\textsuperscript{18}Wang et al. 2004, \textit{IEEE transactions on image processing}. 
Results

Comparison of the Sum-Of-Squares

<table>
<thead>
<tr>
<th>Reference</th>
<th>IFT</th>
<th>GL</th>
<th>sGL</th>
<th>OSCAR</th>
<th>$\ell_1$-ESPIRiT</th>
</tr>
</thead>
</table>

![Reference images](image1)

![IFT images](image2)

![GL images](image3)

![sGL images](image4)

![OSCAR images](image5)

![$\ell_1$-ESPIRiT images](image6)
Results
Quantitative assessment

Comparison between coil images:

Figure: Assessment of the SSIM score per channel.
Comparison between coil images:

Figure: Assessment of the SSIM score per channel.
Results
Comparison of the image channels: low-SNR channel

<table>
<thead>
<tr>
<th>Method</th>
<th>SSIM</th>
</tr>
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<tbody>
<tr>
<td>No reg.</td>
<td>0.630</td>
</tr>
<tr>
<td>Group-LASSO</td>
<td>0.680</td>
</tr>
<tr>
<td>Sparse GL</td>
<td>0.672</td>
</tr>
<tr>
<td>OSCAR</td>
<td>0.646</td>
</tr>
</tbody>
</table>

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Results
Quantitative assessment

Comparison between coil images:

Figure: Assessment of the SSIM score per channel.
Results

Comparison of the image channels: high-SNR channel

<table>
<thead>
<tr>
<th>Method</th>
<th>SSIM</th>
</tr>
</thead>
<tbody>
<tr>
<td>No reg.</td>
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</tr>
<tr>
<td>Group-LASSO</td>
<td>0.880</td>
</tr>
<tr>
<td>Sparse GL</td>
<td>0.863</td>
</tr>
<tr>
<td>OSCAR</td>
<td>0.893</td>
</tr>
</tbody>
</table>

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Conclusion & Outlook

Conclusion:
- New parallel CS-MRI reconstruction algorithm
- No sensitivity maps
- OSCAR outperforms group-LASSO and sparse group-LASSO
- OSCAR and $\ell_1$-ESPIRiT are comparable, however the latest is self-calibrating
- Same optimization method to solve calibration-less MR reconstruction

Perspectives:
- Extension to 3D-MRI
- Study motion impact on the reconstruction

Code is available on 🌐: LElgueddari/pysap/calibrationless_p_mri_reconstruction
Acknowledgement

This project have been granted by the mobility grant of the SFRMBM and the FLI society


References II


