Calibrationless OSCAR-based Image Reconstruction in Compressed Sensing Parallel MRI

L. El Gueddari $^{1,2}$, P. Ciuciu$^{1,2}$, E. Chouzenoux $^{3,4}$, A. Vignaud $^1$ and J-C. Pesquet$^3$

$^1$CEA/NeuroSpin, Gif-sur-Yvette, France

$^2$INRIA-CEA Saclay Ile-de-France, Parietal team, Univ Paris-Saclay, France

$^3$CVN, Centrale-Supélec, Univ. Paris-Saclay, France

$^4$LIGM, Paris-Est University, France

ISBI, 2019, Venice, Italy
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   - Why non-Cartesian acquisition
   - Non-Cartesian MR image reconstruction in parallel imaging

2 Calibration-less MR image reconstruction
   - Problem statement
   - Joint sparsity regularization

3 Experiments & Results
   - Experimental set-up
   - Results

4 Conclusion & Discussion
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Conclusion & Discussion
Anatomical MRI is generally acquired using Cartesian sampling.

Figure: Typical (a) Cartesian (b) parallel acquisition (c) CAIPIRINHA\(^1\) acquisition

... however in some cases non-Cartesian trajectories are useful ...

\(^1\)Breuer et al. 2006, *Magnetic Resonance in Medicine.*
Non-Cartesian trajectories for anatomical MRI

A non-exhaustive list of usage

- For ultra-short echo time imaging\(^2\)
- X-nuclei imaging (TPI\(^3\))
- To correct for motion, especially for abdominopelvic MRI\(^4\)

**Figure:** Stack of stars used for VIBE acquisition

Non-Cartesian trajectories for anatomical MRI

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Figure: A: Free-breathing stack-of-stars VIBE, B: Breath-holding conventional VIBE

Non-Cartesian trajectories for anatomical MRI

Renewed interest to speed-up acquisition in the context of Compressed Sensing\textsuperscript{5}.

\textbf{Figure:} Example of non-Cartesian trajectories.

\textsuperscript{5}Lazarus et al. 2019, \textit{Magnetic Resonance in Medicine}. 

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OSCAR-based p-MRI reconstruction  

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Non-Cartesian trajectories for anatomical MRI

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**Figure**: Comparison of different acquisition trajectories with 16-fold accelerated acquisition on T2*-weighted images.

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Parallel imaging acquisition: collect multiple k-space data using a multi-receiver coil as the latter is known to boost the SNR.

Illustration of multi-receiver coil (phased array).
How do we reconstruct MR images from non-Cartesian k-space measurements in parallel imaging?
Non-Cartesian MR image reconstruction in parallel imaging

Self-calibrating methods

Non-Cartesian reconstruction techniques can be split in two categories:

1. Self-calibrating methods:

\[ \hat{x} = \arg \min_{x \in \mathbb{C}^{N \times L}} \frac{1}{2} \sum_{\ell = 1}^{L} \sigma_{\ell}^{-2} \| F_{\Omega} S_{\ell} x - y_{\ell} \|_{2}^{2} + \lambda \| \Psi x \|_{1} \]  

Note: Extraction of coil sensitivity maps is challenging in non-Cartesian case

\(^7\) Samsonov et al. 2004, *Magnetic Resonance in Medicine*.  

OSCAR-based p-MRI reconstruction
Non-Cartesian MR image reconstruction in parallel imaging

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Non-Cartesian MR image reconstruction in parallel imaging

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\[
\hat{x} = \arg \min_{x \in \mathbb{C}^N} \frac{1}{2} \sum_{\ell=1}^{L} \sigma_\ell^{-2} \| F_\Omega S_\ell x - y_\ell \|_2^2 + \lambda \| \Psi x \|_1
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Non-Cartesian MR Image reconstruction from multi-channel array coil acquisition

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- **Calibration-less methods:**
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  - use the redundant information given by each coil to impose constraints such as low-rank CLEAR\(^9\) or group-sparsity CALM\(^{10}\)

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\(^{10}\) Majumdar and Ward 2012, *Magnetic Resonance in Medicine.*
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  - more likely to be used for on-line image reconstruction

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Problem statement

Calibration-less MR image reconstruction problem solved using an analysis formulation:

Definition

MR image reconstruction is formulated as follows:

\[
\hat{x} = \arg \min_{x \in \mathbb{C}^{N \times L}} \left\{ \frac{1}{2} \sum_{\ell=1}^{L} \sigma_{\ell}^{-2} \| F_{\Omega} x_{\ell} - y_{\ell} \|_{2}^{2} + g(T x) \right\}, \tag{2}
\]

with:

- \( y_{\ell} \in \mathbb{C}^{M} \) the \( \ell \text{th} \) channel-specific data set
- \( x_{\ell} \in \mathbb{C}^{N} \) the \( \ell \text{th} \) channel-specific reconstructed image (ex. \( N = 512 \times 512 \))
- \( F_{\Omega} \) is the forward under-sampling Fourier operator
- \( T \in \mathbb{C}^{N_{\Psi} \times N} \) linear operator related to a sparse decomposition
- \( g \) is a convex regularization term that promotes sparsity
We aim to find:

\[
\hat{x} \in \arg\min_{x \in \mathbb{C}^{N \times L}} \left[ f(x) + g(Tx) \right]
\]  

(3)

where:

- \( f \) is convex, differentiable on \( \mathbb{C}^{N \times L} \) and its gradient is \( \beta \)-Lipschitz
- \( g \in \Gamma_0(\mathbb{C}^{N \times L}) \) with a closed form proximity operator, given by:

\[
\text{prox}_g(z) = \arg\min_{v \in \mathbb{C}^{N \times L}} \frac{1}{2} \| z - v \|^2 + g(v)
\]  

(4)

Note: Those are standard assumption in optimization based image reconstruction methods.
Optimization algorithm

Condat-Vũ sequence

Using a primal-dual optimization method proposed by Condat-Vũ\textsuperscript{11,12}:

\textbf{Algorithm 1: Condat-Vũ algorithm}

\begin{itemize}
  \item initialize $k = 0$, $\tau > 0$, $\kappa > 0$, $x_0$, $z_0$;
  \item while $k \leq K$ do
    \begin{itemize}
      \item $x_{k+1} := x_k - \tau \left( \nabla f(x_k) + T^* z_k \right)$;
      \item $w_{k+1} := z_k + \kappa T \left( 2x_{k+1} - x_k \right)$;
      \item $z_{k+1} := w_{k+1} - \kappa \text{prox}_{g/\kappa} \left( \frac{w_{k+1}}{\kappa} \right)$;
    \end{itemize}
  \item end
\end{itemize}

with:

- the algorithm weakly converges to the solution of Eq. (3) if
  \[
  \frac{1}{\tau} - \kappa \|T\|^2 \geq \frac{\beta}{2}
  \]

- $\tau$ and $\kappa$ hyper-parameters set as follows: $\tau := \frac{1}{\beta}$, $\kappa := \frac{\beta}{2 \|T\|^2}$

\textsuperscript{11}Condat 2013, \textit{Journal of Optimization Theory and Applications}.
\textsuperscript{12}Vũ 2013, \textit{Advances in Computational Mathematics}.
Joint sparsity regularization

Group-LASSO

Parallel imaging has been proved to have tighter recovery guarantees than single channel acquisition when combined with Group-LASSO (GL) regularization\textsuperscript{13}.

Definition

The group-LASSO penalty is defined as follows:

\[
g_{\text{GL}}(\mathbf{z}) = \| \mathbf{z} \|_{1,2} = \sum_{s=1}^{S} \left( \lambda \gamma^s \sqrt{ \sum_{p=1}^{P_s} \sum_{\ell=1}^{L} |z_{sp\ell}|^2 } \right)
\]

- $\lambda$ and $\gamma$ are positive hyper-parameters
- $s$ models the scale or subband dependence

\textsuperscript{13} Chun, Adcock, and Talavage 2016, \textit{IEEE Transactions on Medical Imaging}.

For $\gamma = 1$ the algorithm corresponds to Majumdar and Ward, \textit{Magnetic Resonance in Medicine}, 2012.
Joint sparsity regularization
Sparse group-LASSO

**Variant:** Sparse group-LASSO\(^{15}\) (sGL)

**Definition**

\[
\forall \mathbf{z} \in \mathbb{C}^{N_{\psi} \times L}, g_{\text{sGL}}(\mathbf{z}) = g_{\text{GL}}(\mathbf{z}) + \mu \|\mathbf{z}\|_1 \quad (5)
\]

- \(\mu\) being positive hyper-parameter.

sGL proximity operator\(^{18}\) is closed form and corresponds to the composition of GL proximity operator with soft-thresholding.

Joint sparsity regularization
Octagonal Shrinkage and Clustering Algorithm for Regression

Inferring the structure via a pairwise $\ell_\infty$ norm.

OSCAR regularization:\textsuperscript{16}:

\textbf{Definition}

\[ g_{\text{OSCAR}}(z) = \sum_{s=1}^{S} \lambda \left[ \sum_{j=1}^{P_s L} |z_{sj}| + \gamma \sum_{j<k} \max\{ |z_{sj}|, |z_{sk}| \} \right] \]

\[ = \sum_{s=1}^{S} \lambda \left[ \sum_{j=1}^{P_s L} (\gamma (j - 1) + 1) |z_{sj}| \right] \]

(6)

where:
- $z_\downarrow \in \mathbb{C}^{N_\Psi \times L}$ the wavelet coefficients sorted in decreasing order, i.e.:
  \[ \forall s \in \mathbb{N}, |z_{s1}| \leq \cdots \leq |z_{sP_s L}|. \]
- $\lambda$ and $\gamma$ are some positive hyper-parameters that need to be set

\textsuperscript{16}Bondell and Reich 2008, Biometrics.
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Experimental set-up

Sequence parameters:
- Ex-vivo baboon brain
- 7T Siemens Scanner GRE
- 1Tx/32Rx Nova coil
- Sparkling trajectory
- \(390 \mu m \times 390 \mu m\) in plane-resolution
- 3mm slice thickness
- Acceleration factor of 15 in time
- Under-sampling factor of 2.5
- \(T\): Undecimated Bi-Orthogonal 7-9 wavelet transform

Hyper-parameters set using a grid-search procedure.
Cartesian scan \(512 \times 512\) was acquired and used for reference.

Figure: Sparkling trajectory
Results
Quantitative assessment

Coil combination: Square root of the Sum-Of-Squares
Structural SIMilarity Index (SSIM)\textsuperscript{17} used to set hyper-parameters

<table>
<thead>
<tr>
<th></th>
<th>SSIM</th>
<th>pSNR (dB)</th>
<th>NRMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>IFT</td>
<td>0.847</td>
<td>26.50</td>
<td>0.263</td>
</tr>
<tr>
<td>GL</td>
<td>0.864</td>
<td>26.92</td>
<td>0.254</td>
</tr>
<tr>
<td>sGL</td>
<td>0.851</td>
<td>26.77</td>
<td>0.259</td>
</tr>
<tr>
<td>OSCAR</td>
<td>0.875</td>
<td>30.49</td>
<td>0.177</td>
</tr>
<tr>
<td>$\ell_1$-ESPIRiT</td>
<td>0.874</td>
<td>28.32</td>
<td>0.238</td>
</tr>
</tbody>
</table>

Table: Image quality assessment for all regularizers.

Note: $\ell_1$-ESPIRiT is a self-calibrating method.

\textsuperscript{17}Wang et al. 2004, *IEEE transactions on image processing*.
Results

Comparison of the Sum-Of-Squares

Reference  IFT  GL  sGL  OSCAR  $\ell_1$-ESPIRiT
Results
Quantitative assessment

Comparison between coil images:

Figure: Assessment of the SSIM score per channel.
Comparison between coil images:

**Figure:** Assessment of the SSIM score per channel.
Results

Comparison of the image channels: low-SNR channel

No reg.
SSIM = 0.630

Group-LASSO
SSIM = 0.680

Sparse GL
SSIM = 0.672

OSCAR
SSIM = 0.646
Comparison between coil images:

Figure: Assessment of the SSIM score per channel.
Results

Comparison of the image channels: high-SNR channel

<table>
<thead>
<tr>
<th>Method</th>
<th>SSIM</th>
</tr>
</thead>
<tbody>
<tr>
<td>No reg.</td>
<td>0.846</td>
</tr>
<tr>
<td>Group-LASSO</td>
<td>0.880</td>
</tr>
<tr>
<td>Sparse GL</td>
<td>0.863</td>
</tr>
<tr>
<td>OSCAR</td>
<td>0.893</td>
</tr>
</tbody>
</table>

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Conclusion:
- New parallel CS-MRI reconstruction algorithm
- No sensitivity maps
- OSCAR outperforms group-LASSO and sparse group-LASSO
- OSCAR and $\ell_1$-ESPIRiT are comparable, however the latest is self-calibrating
- Same optimization method to solve calibration-less MR reconstruction

Perspective:
- Extension to 3D-MRI
- Study motion impact on the reconstruction


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OSCAR-based p-MRI reconstruction

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