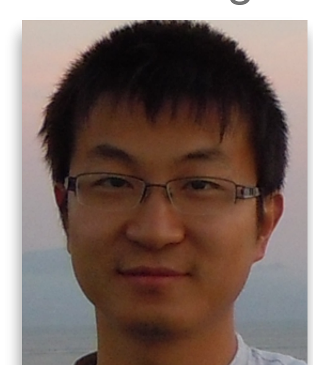
Sparse reconstruction of radio transients and multichannel images

Julien Girard **Jean-Luc Starck**

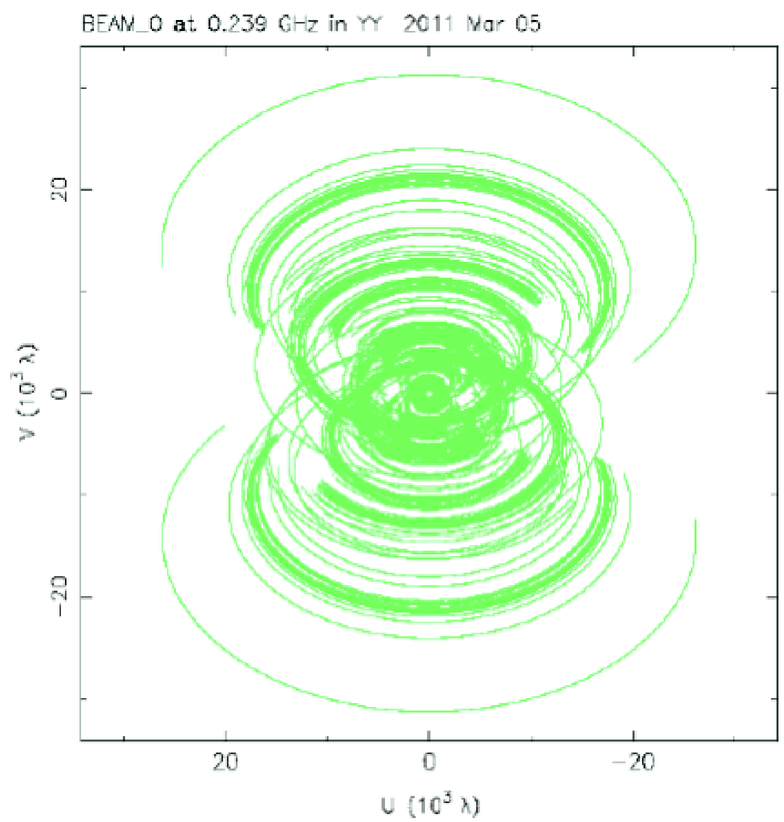
CEA, IRFU, AIM, Service d'Astrophysique, France

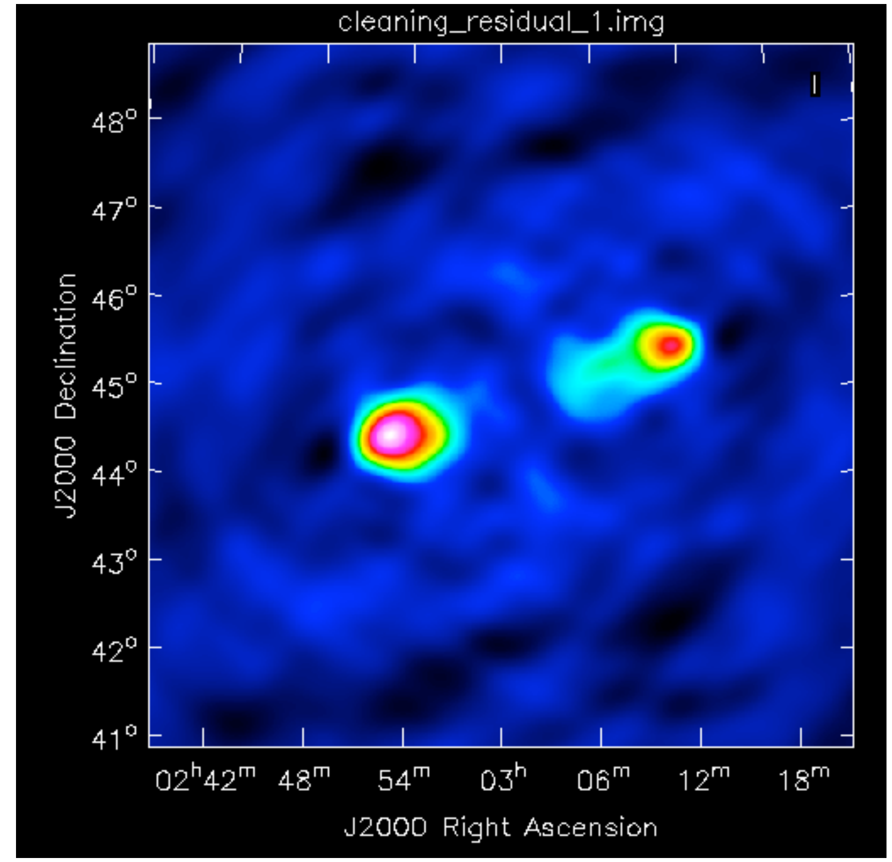
http://jstarck.cosmostat.org

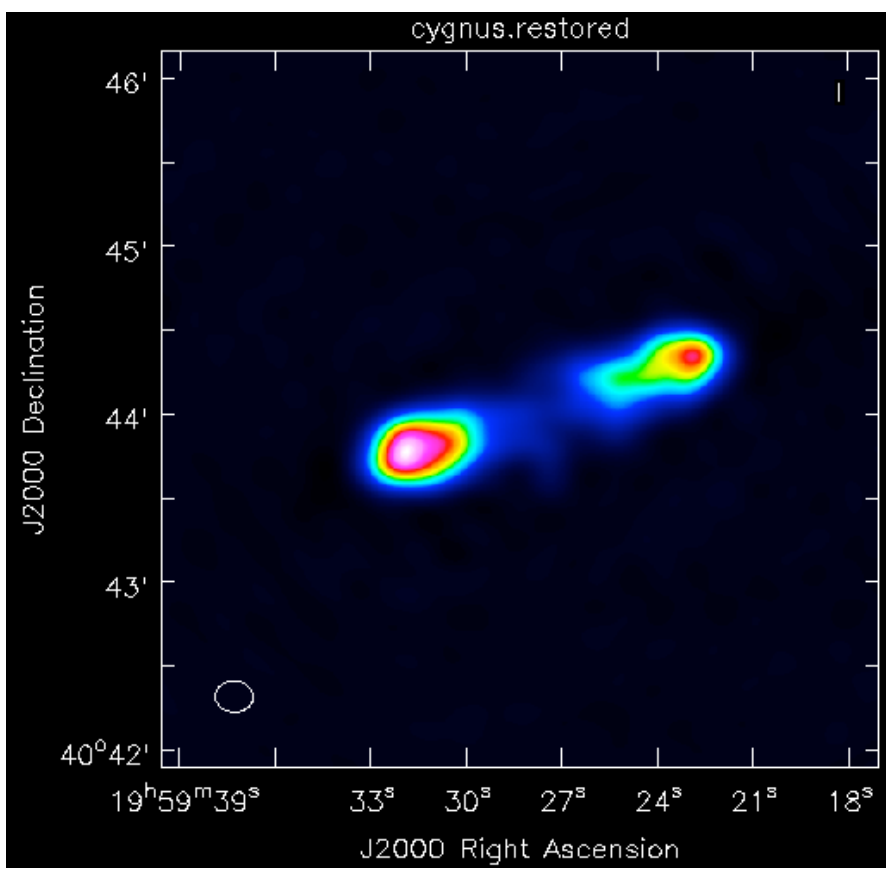
S. Corbel J. Bobin C. Tasse M. Jiang

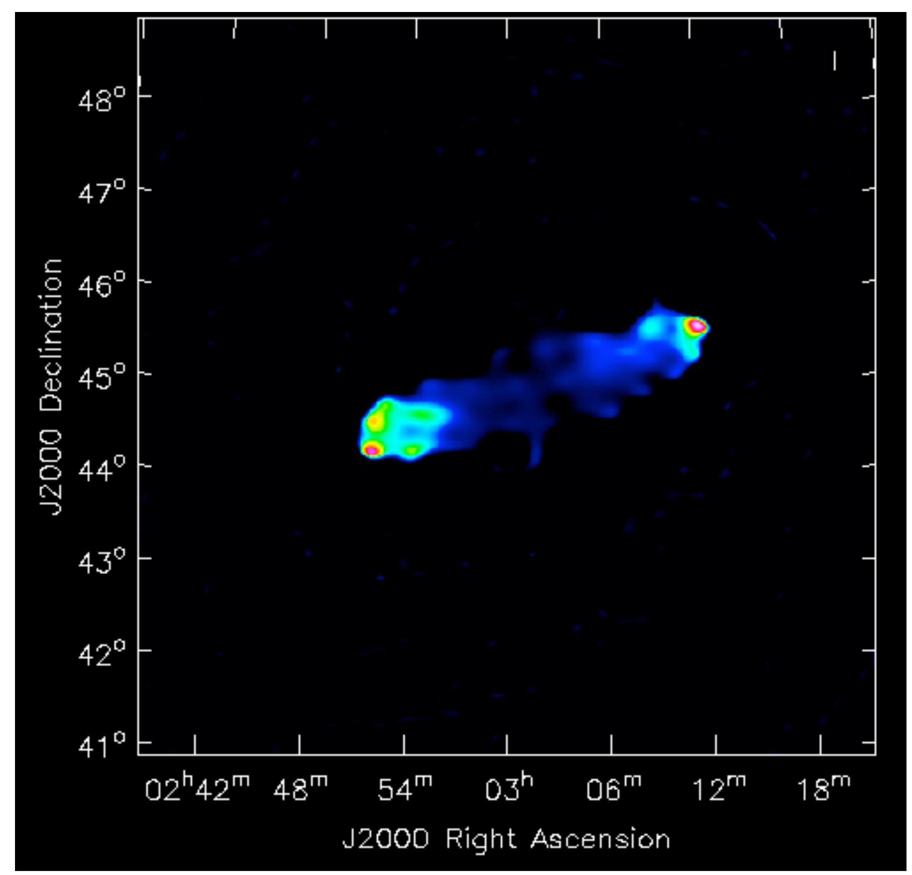
   

Compressed Sensing & LOFAR Cygnus A Data









Garsden et al, “**LOFAR Image Sparse Reconstruction**”, A&A, 575, A90, 2015.

http://arxiv.org/abs/1406.7242

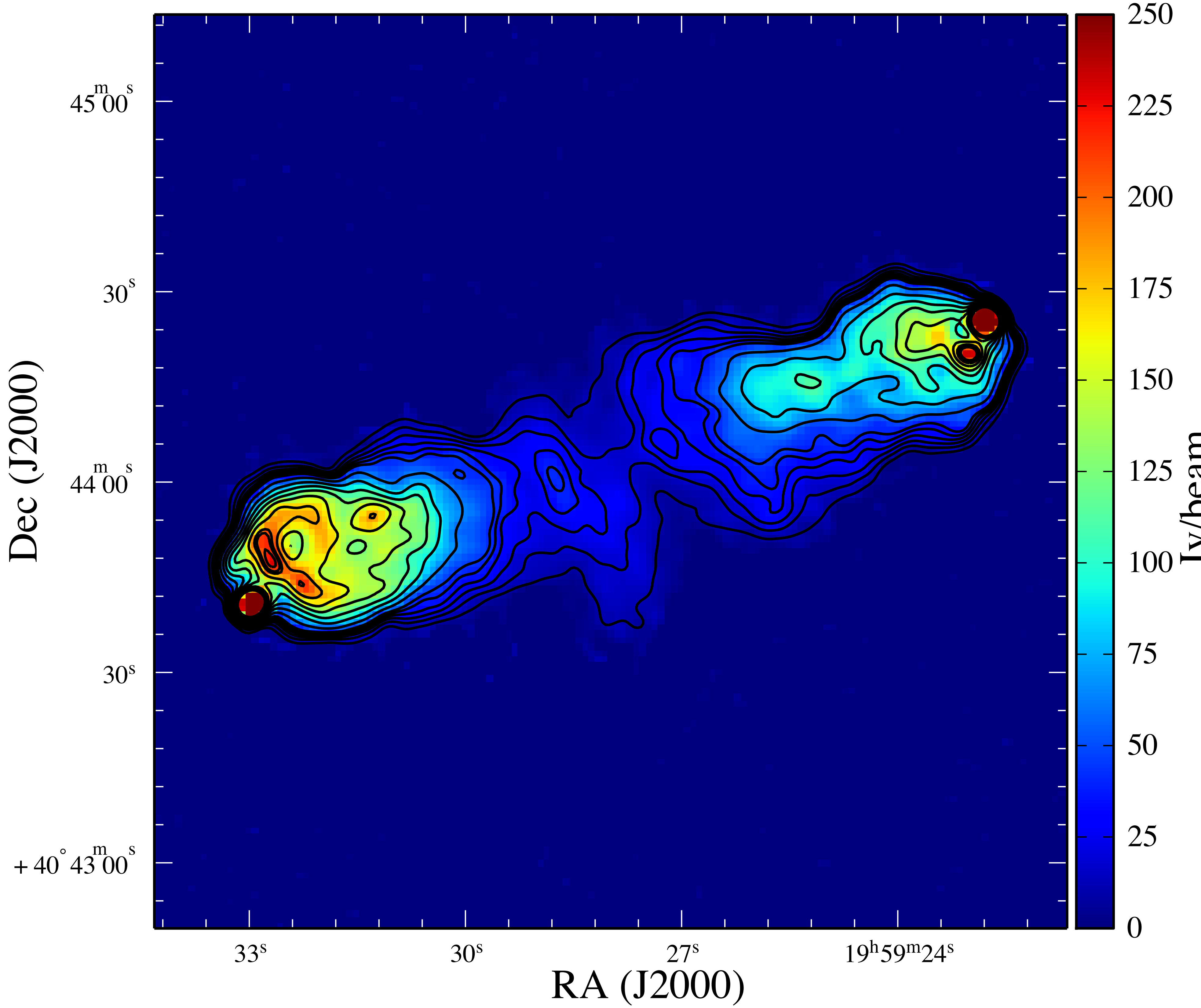
Garsden et al, “LOFAR Image Sparse Reconstruction”, A&A, 575, A90, 2015, ArXiv:1406.7242.

J. Girard

H. Garsden

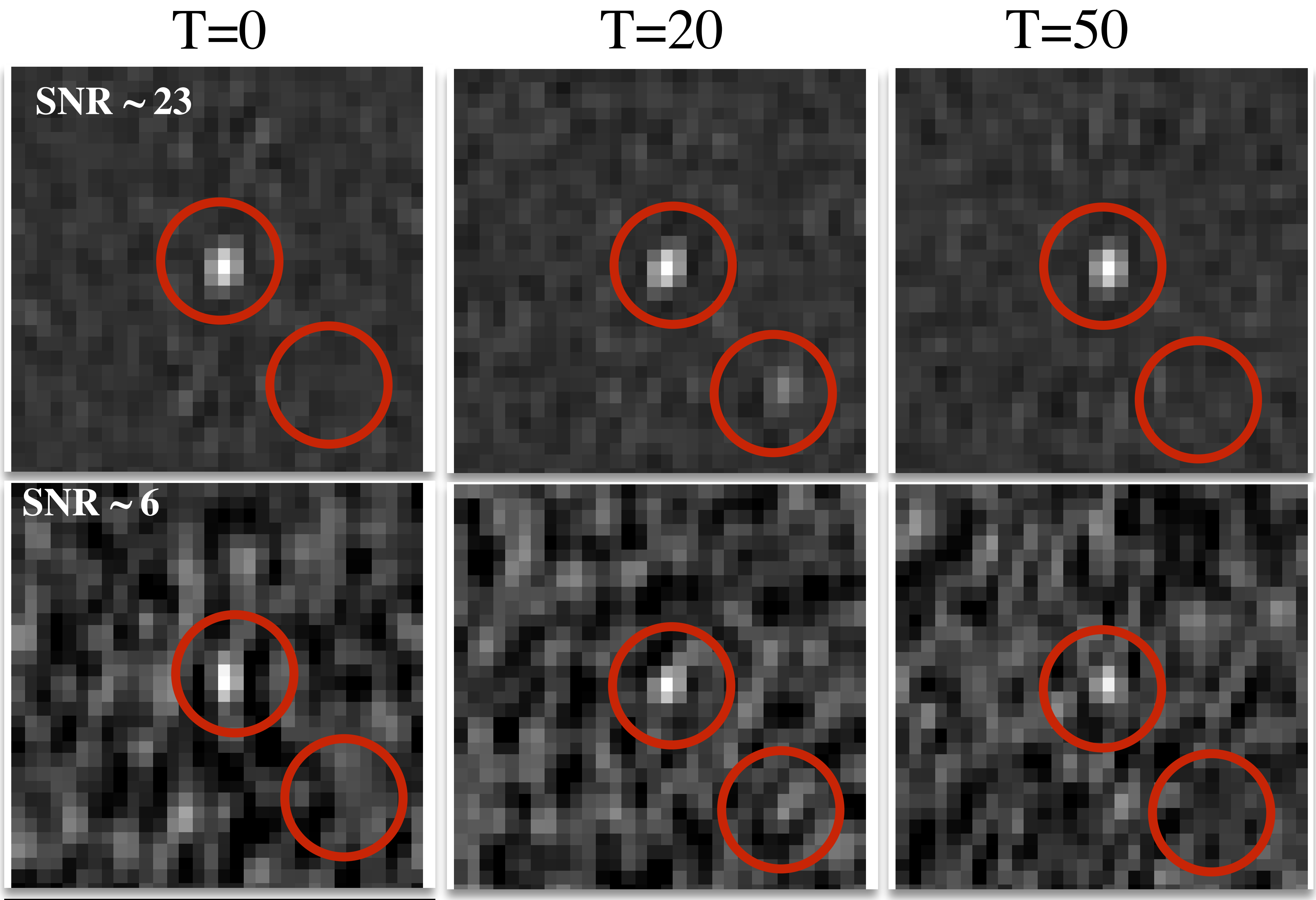
S. Corbel

C. Tasse



Colorscale: reconstructed 512x512 image of Cygnus A at 151 MHz (with resolution 2.8” and a pixel size of 1”). Contours levels are [1,2,3,4,5,6,9,13,17,21,25,30,35,37,40] Jy/Beam from a 327.5 MHz Cyg A VLA image (Project AK570) at 2.5” angular resolution and a pixel size of 0.5”.
Recovered features in the CS image correspond to real structures observed at higher frequencies.

Dirty map



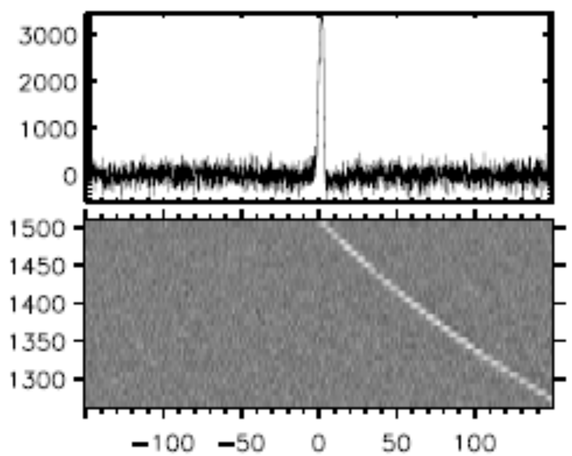
The Transient Universe in Radio

<<1ms 1s 10s 1m 10 m H Days Weeks M Y

Variability timescale

- **Fast transient sources**

  Coherent emission
  Relatively fast variability
  High brightness temperature
  Usually associated with pulsars

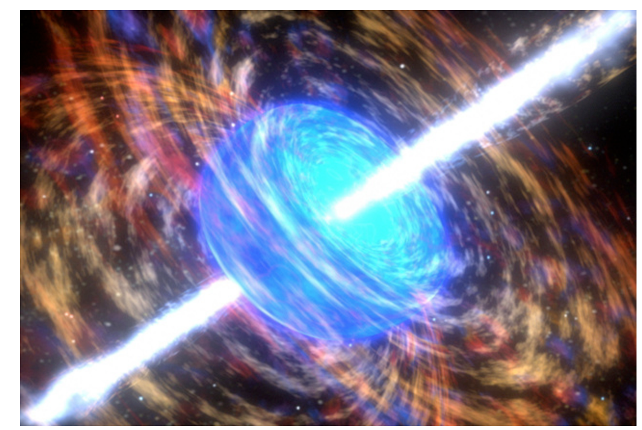


Time series	V(t)
Dynamic spectroscopy	V(f,t)

☺ **Good time resolution**
☹ **No/low angular res.**
☹ **Low instantaneous SNR**
☹ **few samples for imaging**

- **Slow transient sources**

  Incoherent synchrotron emission
  Relatively slow variability
  Brightness temperature limited (10^{12} K)
  Associated with all explosive events



Light curves
data mining

☹ **Poor time resolution**
☺ **Good angular resolution**
☺ **Good integrated SNR**
☺ **enough samples for imaging**

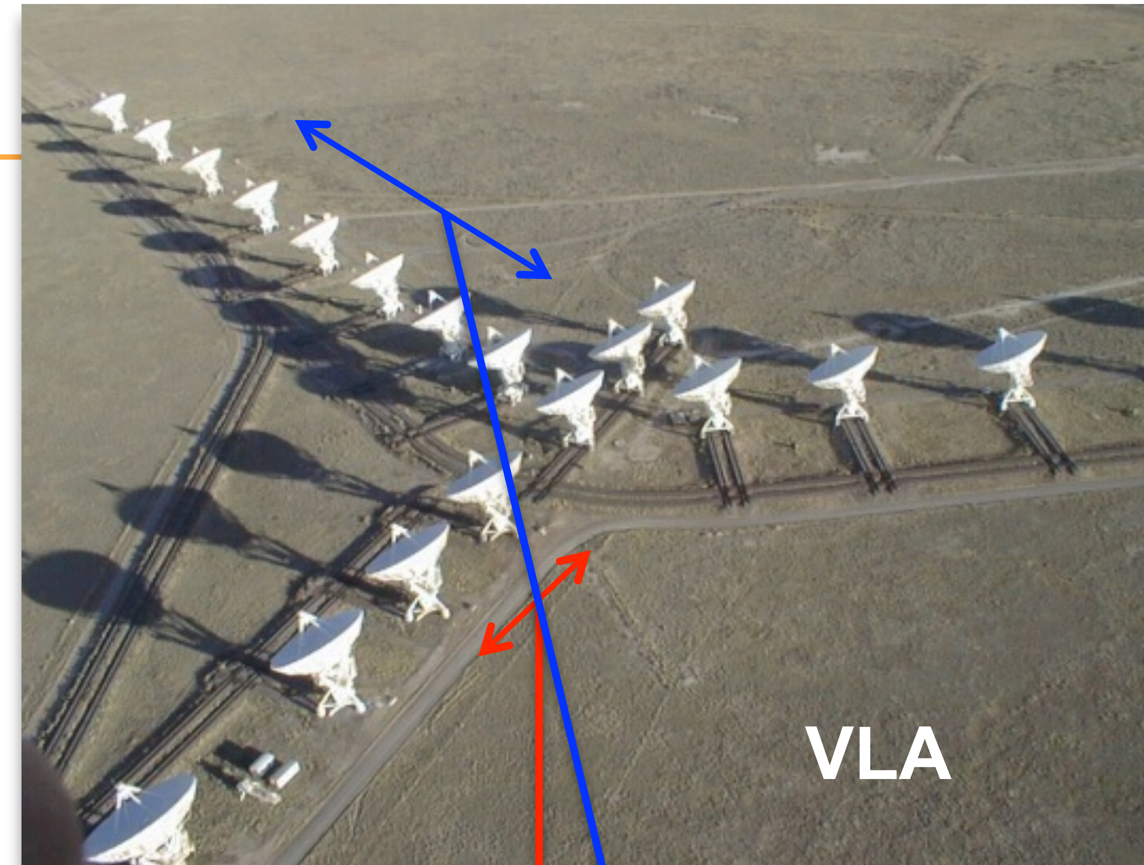
+ others problems (instrument stability, ionosphere...)

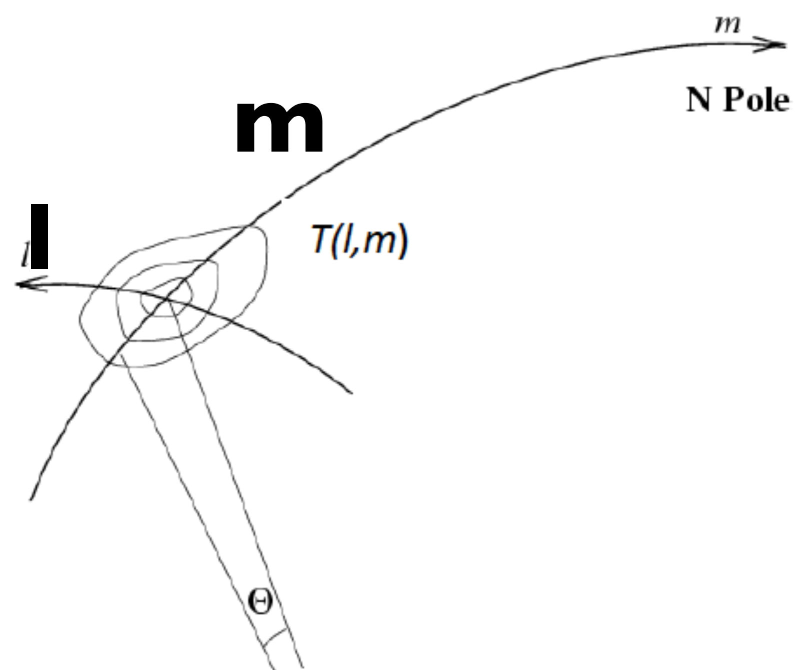
Imaging with interferometry

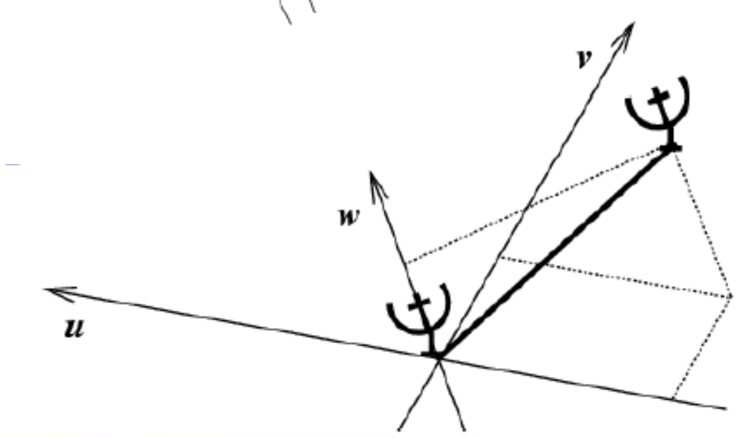
N **antennas/telescopes**

$\frac{N(N-1)}{2}$ **independent baselines**

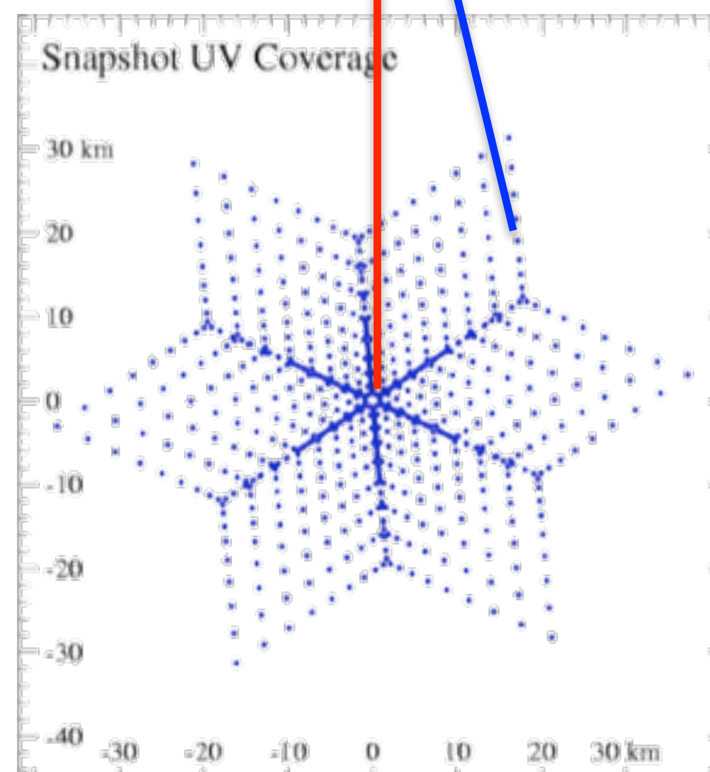
1 projected baseline
= 1 sample in the Fourier « u,v » plane







v



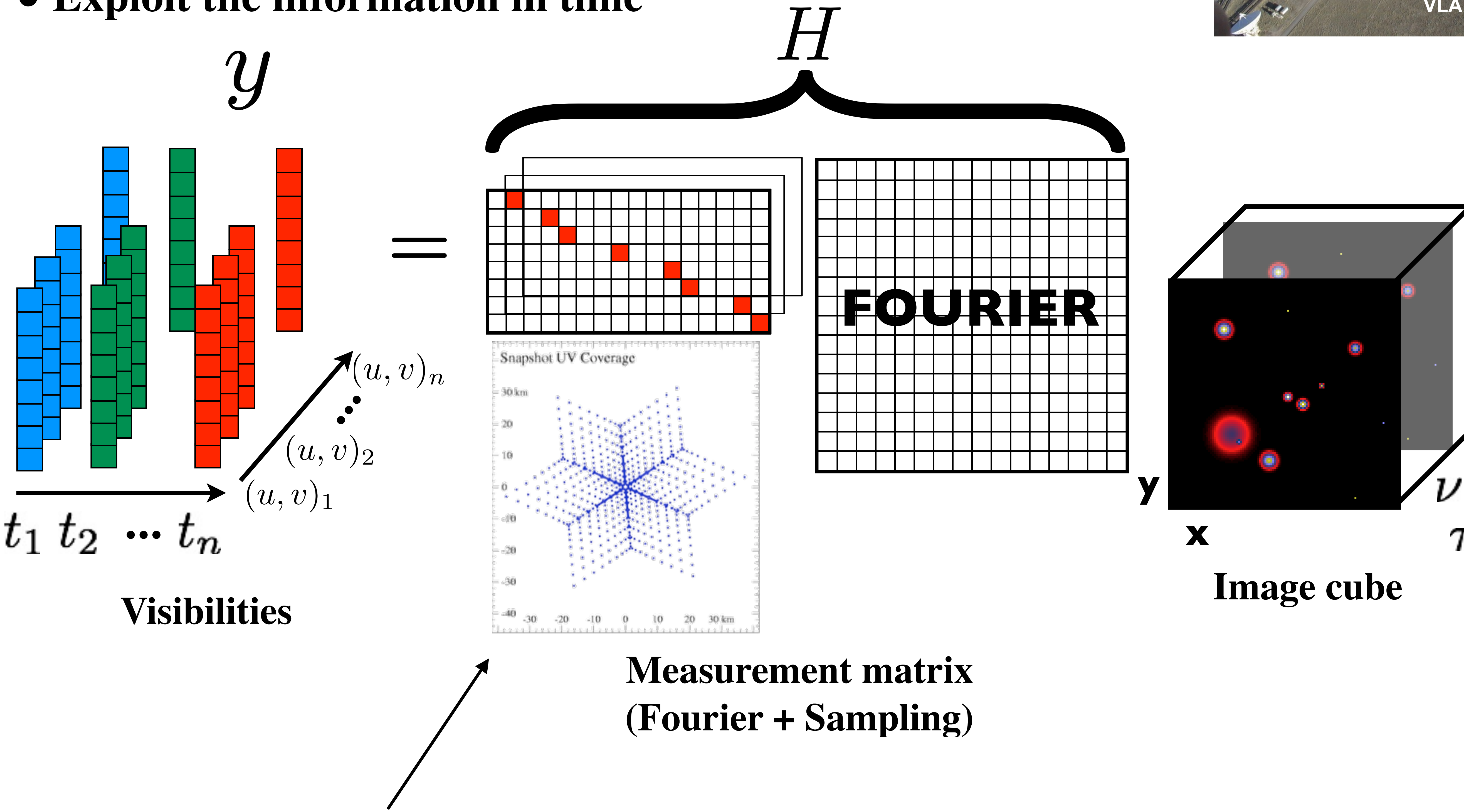
u

(u,v) plane sampling

$$V(u,v) = \int\int T(l,m) e^{-i2\pi(ul+vm)} dl dm \longrightarrow T(l,m)$$

Application of 2D-1D sparse reconstruction in Radio-Interferometry

- **Exploit the information in time**



- **The masking operator will also be time-dependent**

Fundamental Sparse Ingredients

1 - Which Norm ? **l_0 through re-weighted l_1**

2 -Constraint versus Lagrangian formulation ? Lagrangian

3 - Analysis versus Synthesis ? **Analysis**

4 - Which dictionary ? **2D-1D WT**

5 - Which noise model ? Stationary correlated noise

6 - Which minimization method ? **Condat-Vu algorithm is very efficient.**

7 - How to fix the regularization parameter ? Physical interpretation of the regularization parameter, through the noise modeling => fully automatic approach.

We need **dictionaries** - space and time are independent

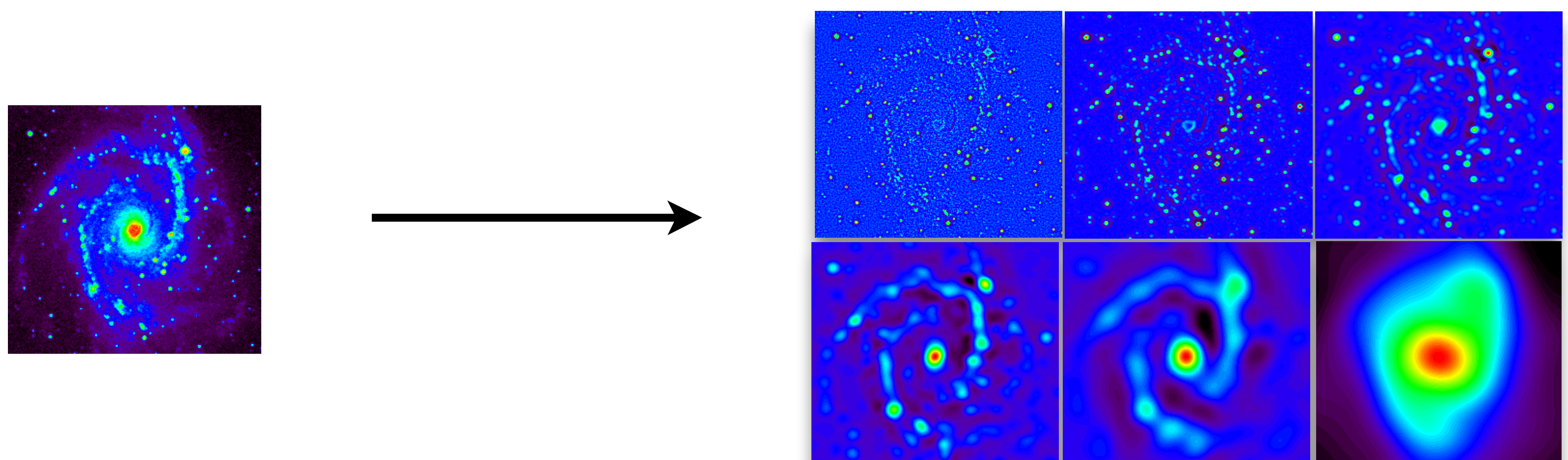
$$\psi(x,y,t) = \psi^{(xy)}(x,y)\psi^{(t)}(t)$$

- **for the 2D spatial signal**

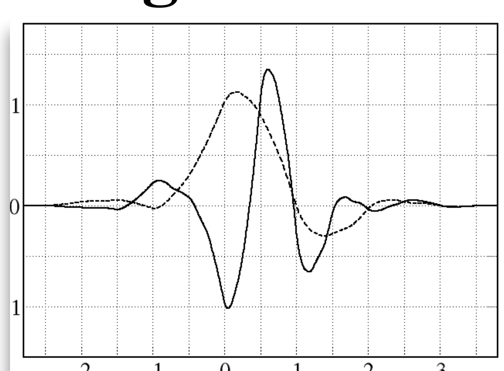
Starlets [Starck et al. 2011]

(Isotropic Undecimated Wavelet Transform)

$$I(k,l) = c_{J,k,l} + \sum_{j=1}^{J} w_{j,k,l}$$



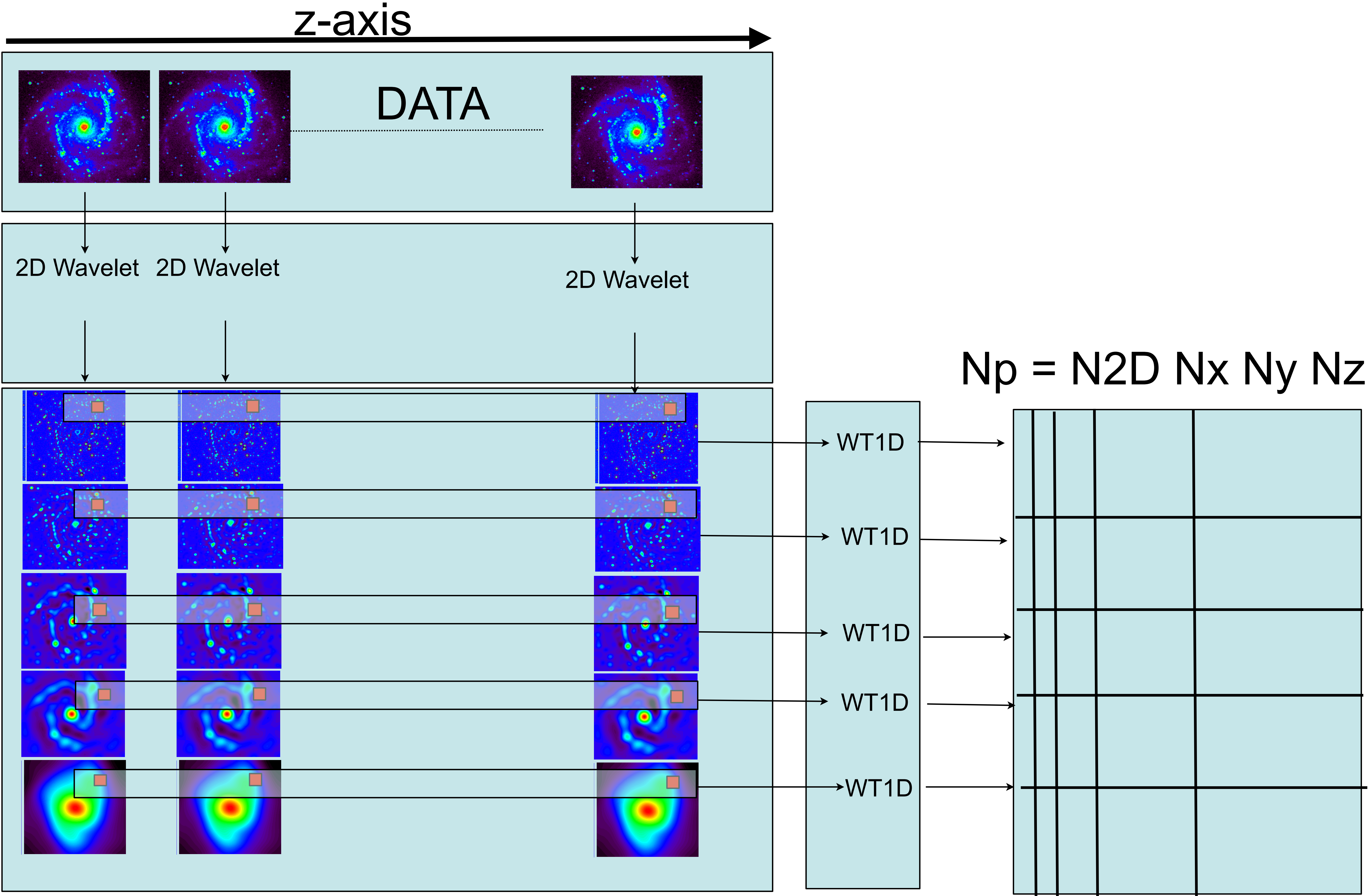
- **for the 1D temporal signal**



7/9 wavelets

2D-1D Wavelet Decomposition

$$w_{j_1,j_2}[k_x,k_y,k_z] = D * \bar{\psi}^{(xy)}_{j_1} * \bar{\psi}^{(z)}_{j_2}(x,y,z)$$

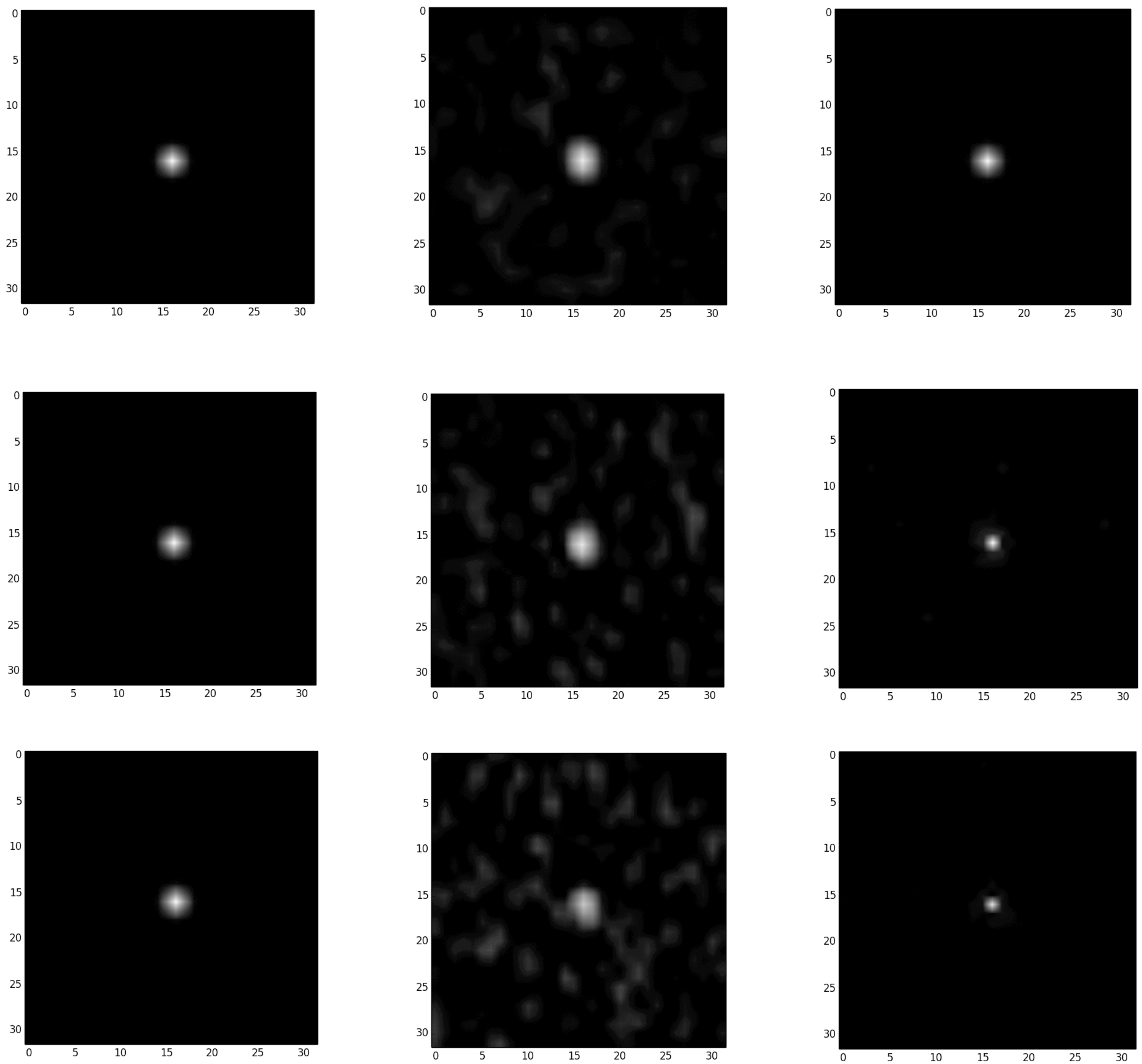


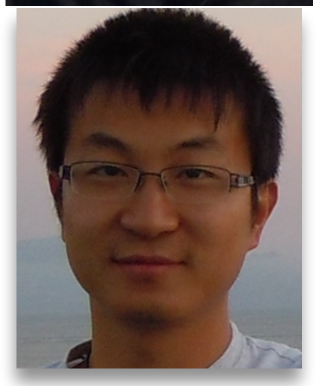
Sky model | Dirty map | Reconstruction

σ=0.5

σ =1.0

σ =1.5





CosmoStat Lab

Girard et al, Sparse spatio-temporal imaging of radio transients, in preparation, 2016

Test #1: SNR - Results

Dirty cubes

Marginal improvement of SNR with time

SNR turn-over below N_t =40

➙ temporal dilution

CLEANed cubes

Higher SNR at low noise

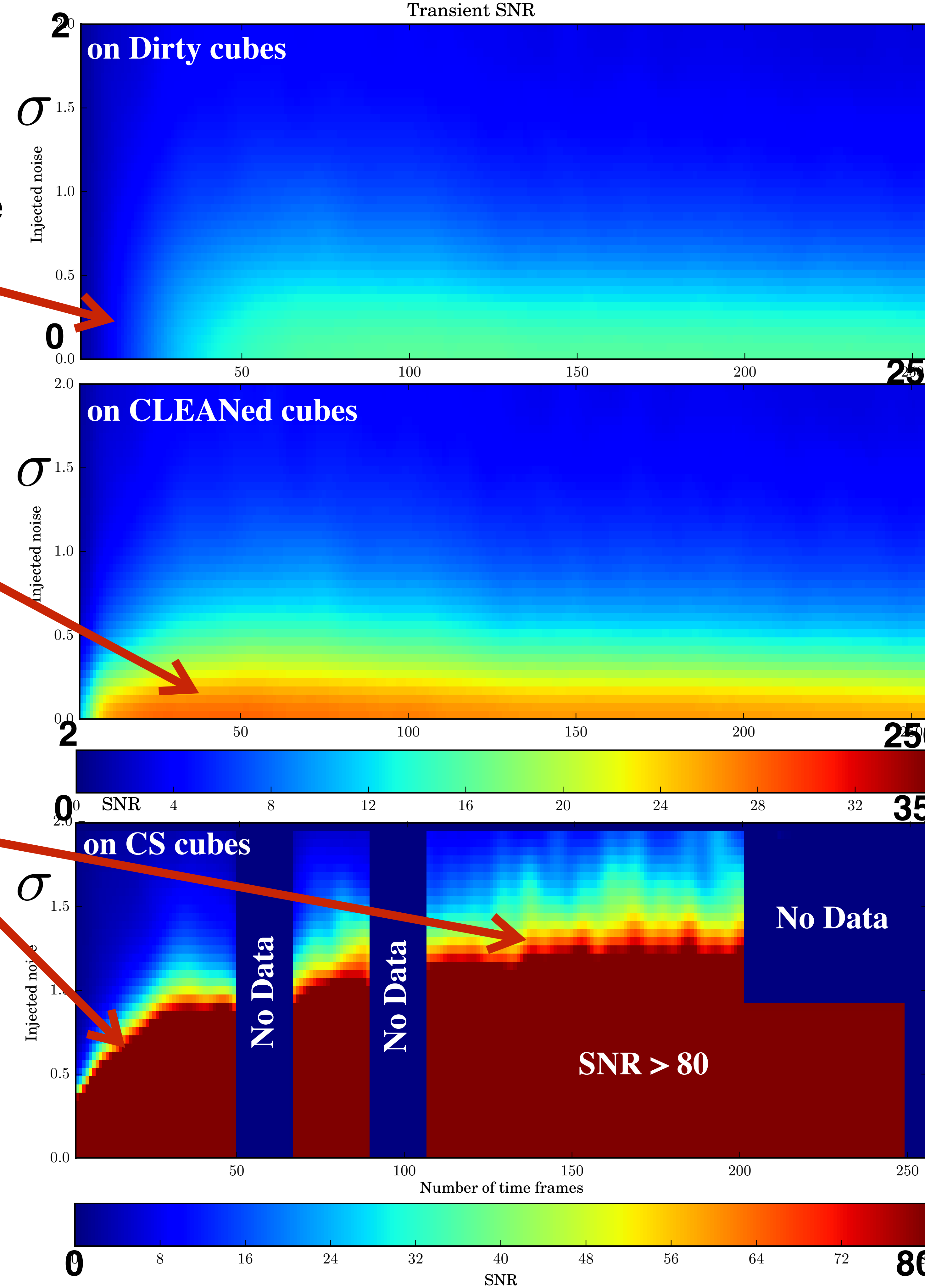
Temporal dilution effect reduced

2D1D Sparse cube

Higher SNR at low & high noise

Slow decrease of SNR due to dilution

One order of magnitude improvement in SNR

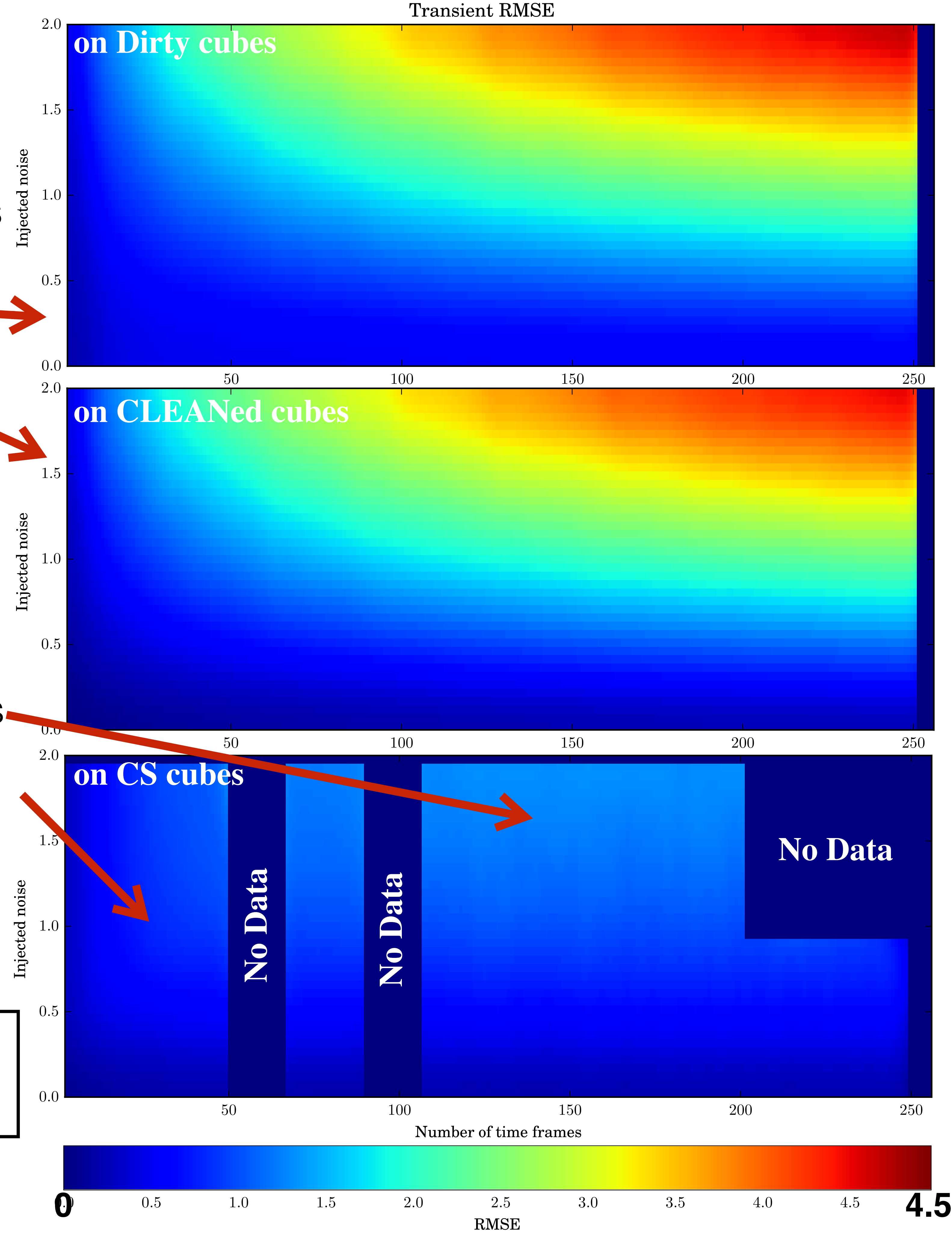


Test #2: Time profile

Dirty & CLEANed cubes

Higher RMSE in high noise snapshots

Similar profile error



2D1D Sparse cube

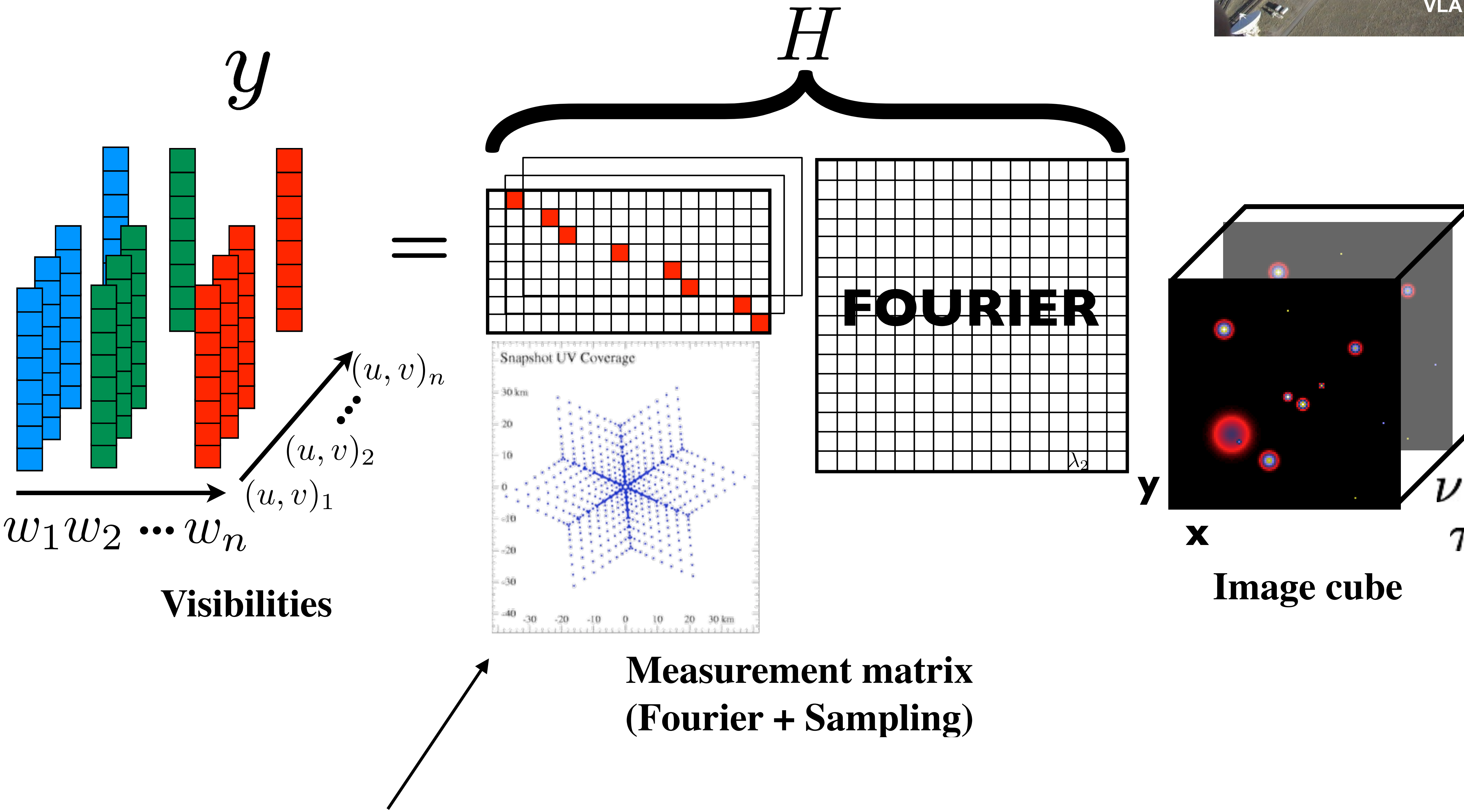
Higher RMSE in high noise snapshots

Better overall profile reconstruction

Factor of ~3 reduction of the RMSE of transient profile

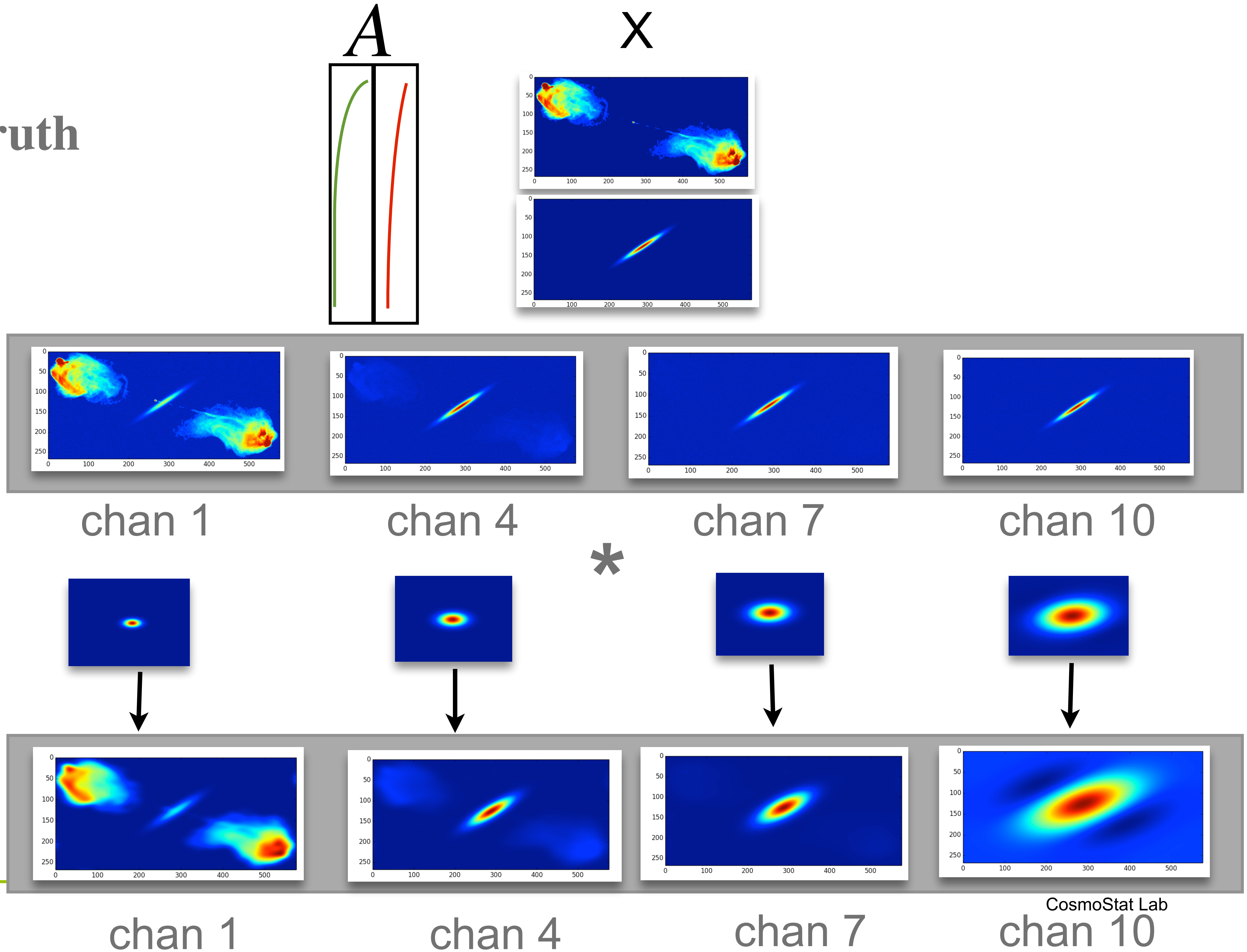
Application of learned sparse reconstruction in Radio-Interferometry

- **Exploit the correlation along the wavelength axis**



- **The masking operator will also be wavelength-dependent**

Hyperspectral Data



Ground Truth

Mixtures

$X = AS$

PSF H

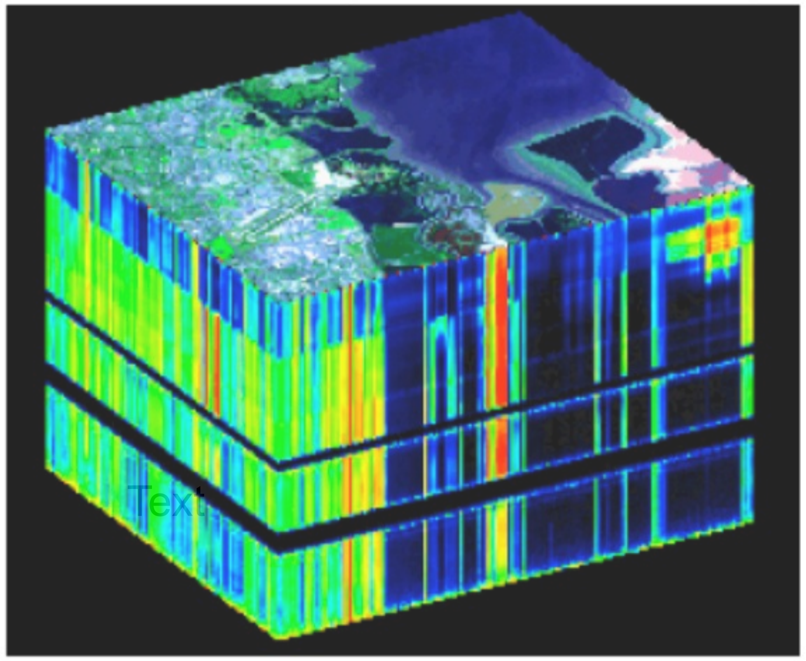
Data

$Y = HX + N$

CosmoStat Lab

Modelisation

Marc Lennon
Département Image et Traitement de l'Information (ITI)
École Nationale Supérieure des Télécommunications de Bretagne



$$Y_i = H_i * \sum_{s=1}^{S} a_{i,s} X_s + N$$

Need to add constraint

$$min_{A,X} = \parallel Y - \mathcal{H}(AX) \parallel^2 \quad s.t. \;\; \mathcal{C}(X, A)$$

Blind Source Separation

$$Y = AX + N$$

Sparse Component Separation: the GMCA Method

X and S are estimated alternately and iteratively in two steps :

1) Estimate X assuming A is fixed (iterative thresholding) :

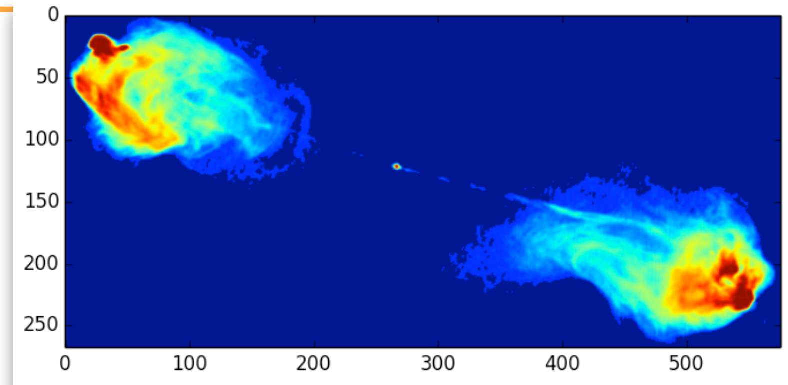
$$\min_X \|Y - AX\|^2_{F,\mathbf{\Sigma}} + \sum_j \lambda_j \|\Phi^t x_j\|_1$$

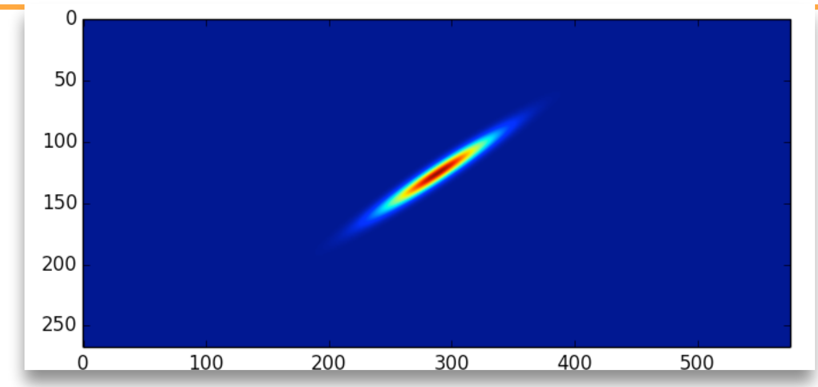
2) Estimate A assuming X is fixed (a simple least square problem) :

$$\min_A \|Y - AX\|^2_{F,\mathbf{\Sigma}}$$

Experiments

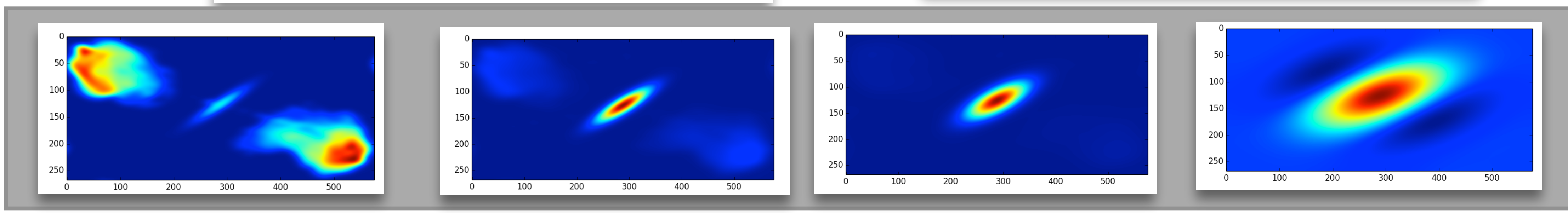
Model sources



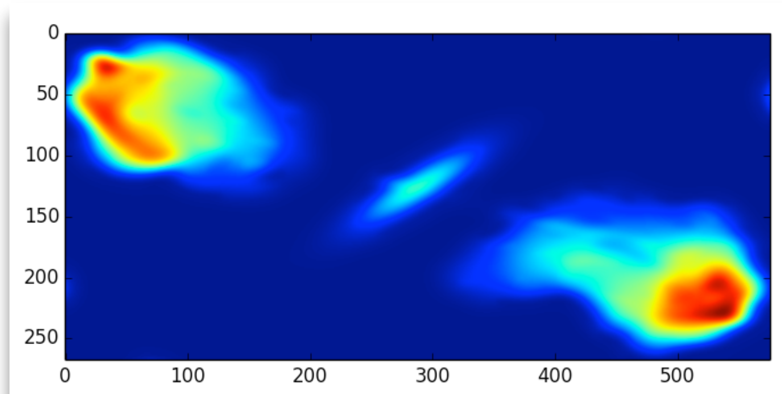


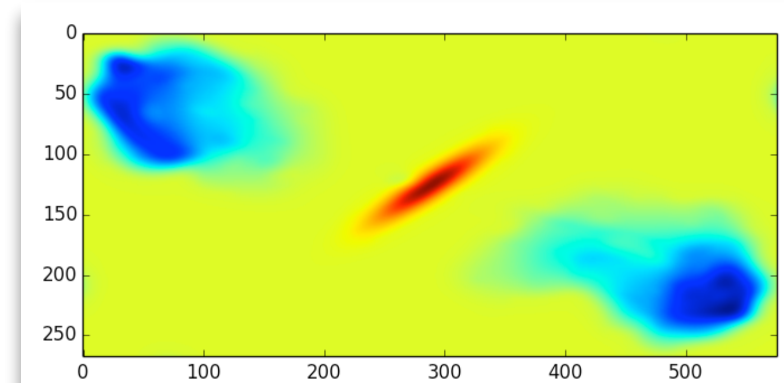
Data

4 out of 10 channels

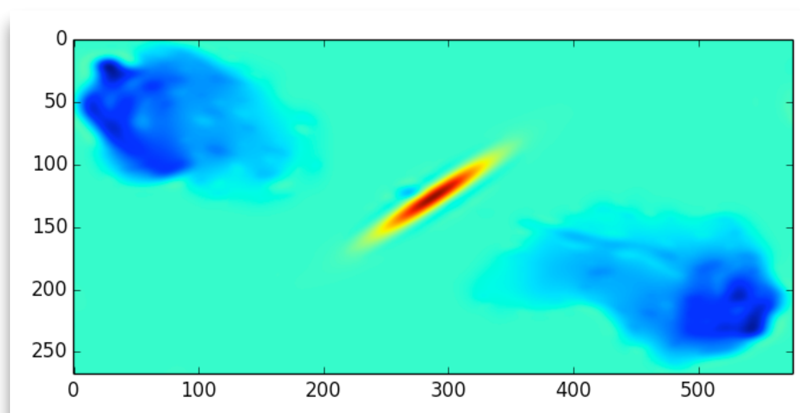


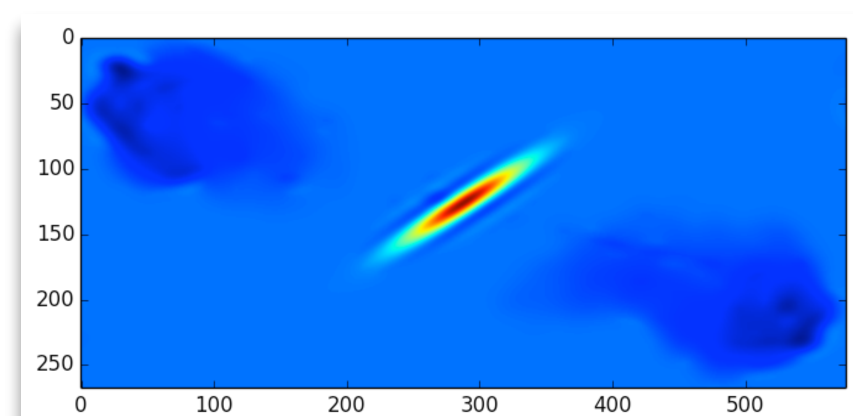
BSS only (GMCA), no deconvolution



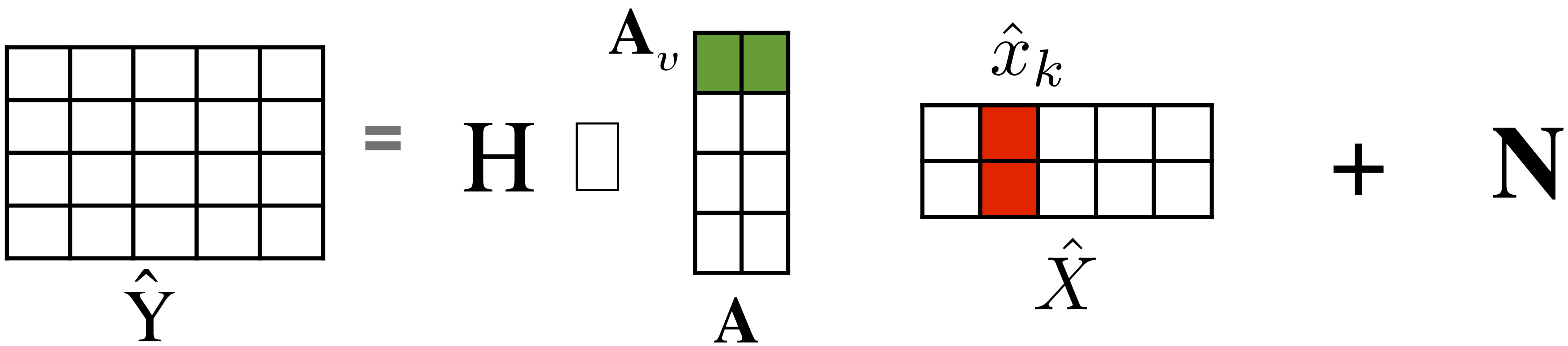


Channel by channel deconvolution (ForWaRD) followed by a BSS (GMCA)





Deconvolved Blind Source Separation (DBSS)



$$\forall i;\ y_i = h_i \star \left(\sum_j a_{ij} x_j \right) + n_i$$

Globally: $\mathbf{Y} = \mathcal{H}\left(\mathbf{AX}\right) + \mathbf{N}$

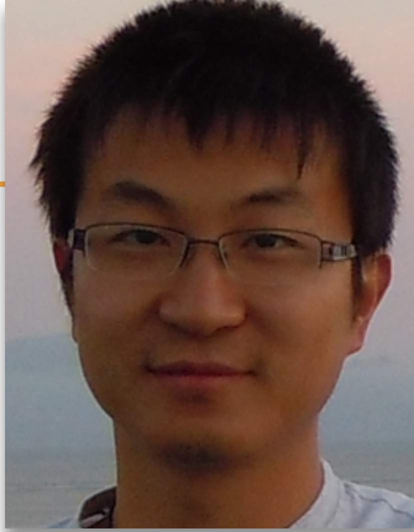
where $\mathcal{H}$ is the multichannel convolution operator

In Fourier space: $\forall i;\ \hat{y}_i = \hat{h}_i \left(\sum_j a_{ij} \hat{x}_j \right) + \hat{n}_i$

Radio-Interferometry: $V = M.A\hat{X} + \hat{N}$

has an infinite number of solutions and non-convex problem.

DecGMCA



Problem formulation

$$\min_{\mathbf{A},\mathbf{X}} \sum_{i}^{\mathrm{N_c}} ||\hat{Y}_i - \hat{h}_i(\widehat{AX})_i||_2^2 + \sum_{i}^{\mathrm{N_s}} \lambda_i ||\mathbf{\Phi}^t \mathbf{X}_i||_0,$$

1) Estimate X assuming A is fixed (iterative thresholding) :

$$\min_{\mathbf{X}} \sum_{i}^{\mathrm{N_c}} ||\hat{Y}_i - \hat{h}_i(\widehat{AX})_i||_2^2 + \sum_{i}^{\mathrm{N_s}} \lambda_i ||\mathbf{\Phi}^t \mathbf{X}_i||_0,$$

Multichannel Tikkhonov Regularization

+ Wavelet Thresholding

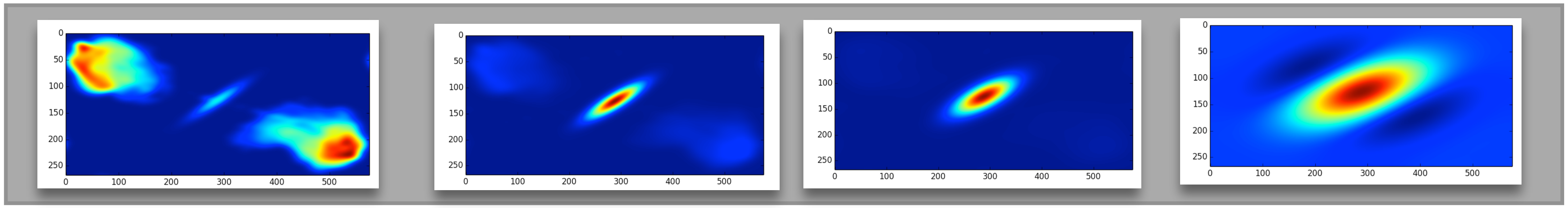
2) Estimate A assuming X is fixed (a simple least square problem) :

$$\min_{\mathbf{A}} \sum_{i}^{\mathrm{N_c}} ||\hat{Y}_i - \hat{h}_i(\widehat{AX})_i||_2^2$$

Ming et al, Joint Multichannel Deconvolution and Blind Source Separation, submitted, 2017.

Experiments(Source reconstruction)

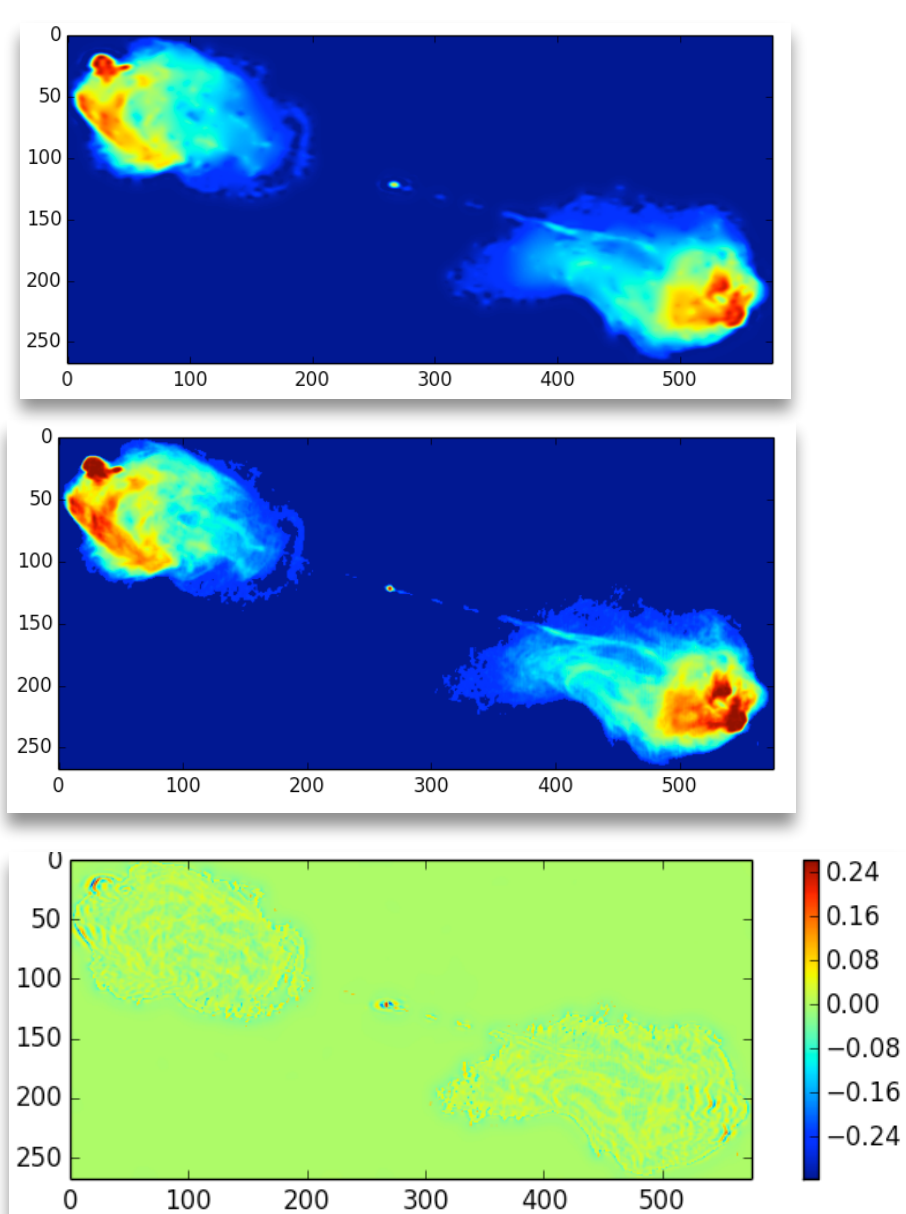
Data

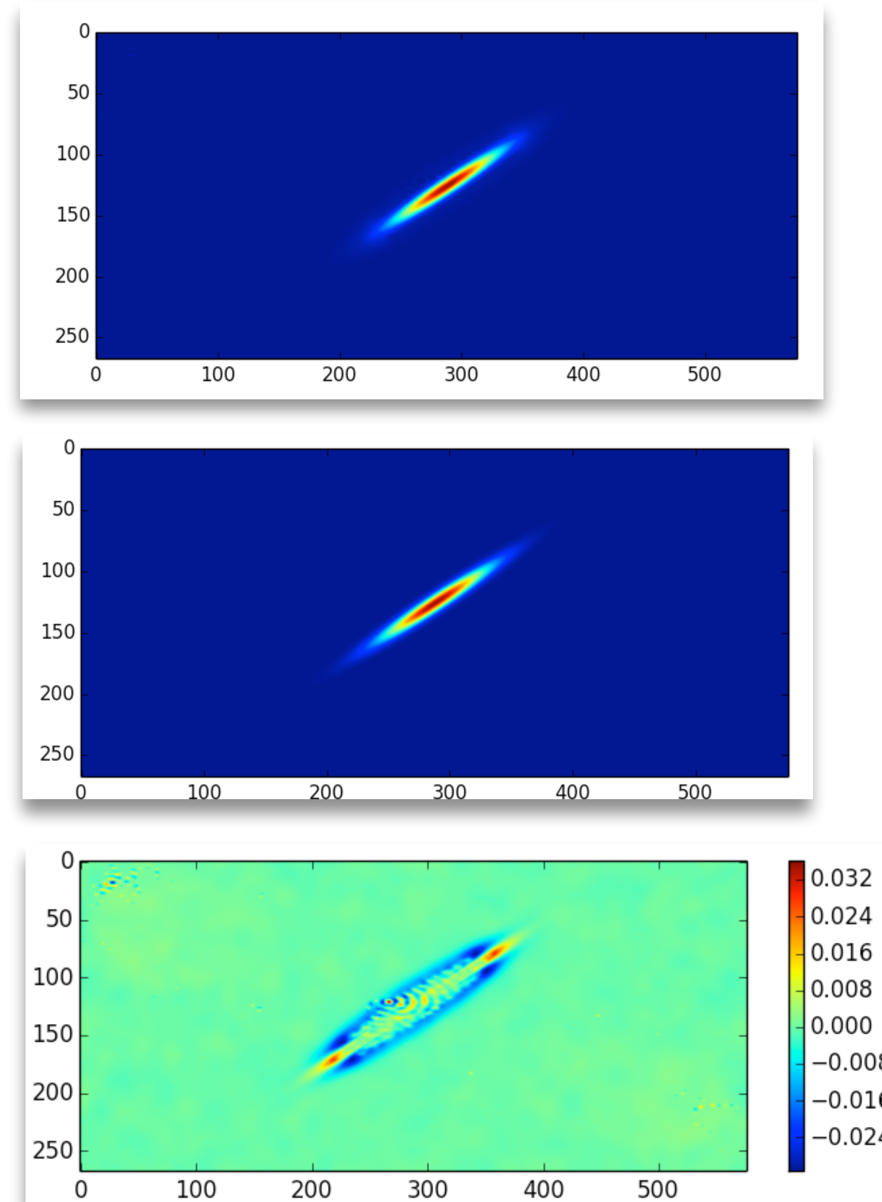


Reconstruction

Truth

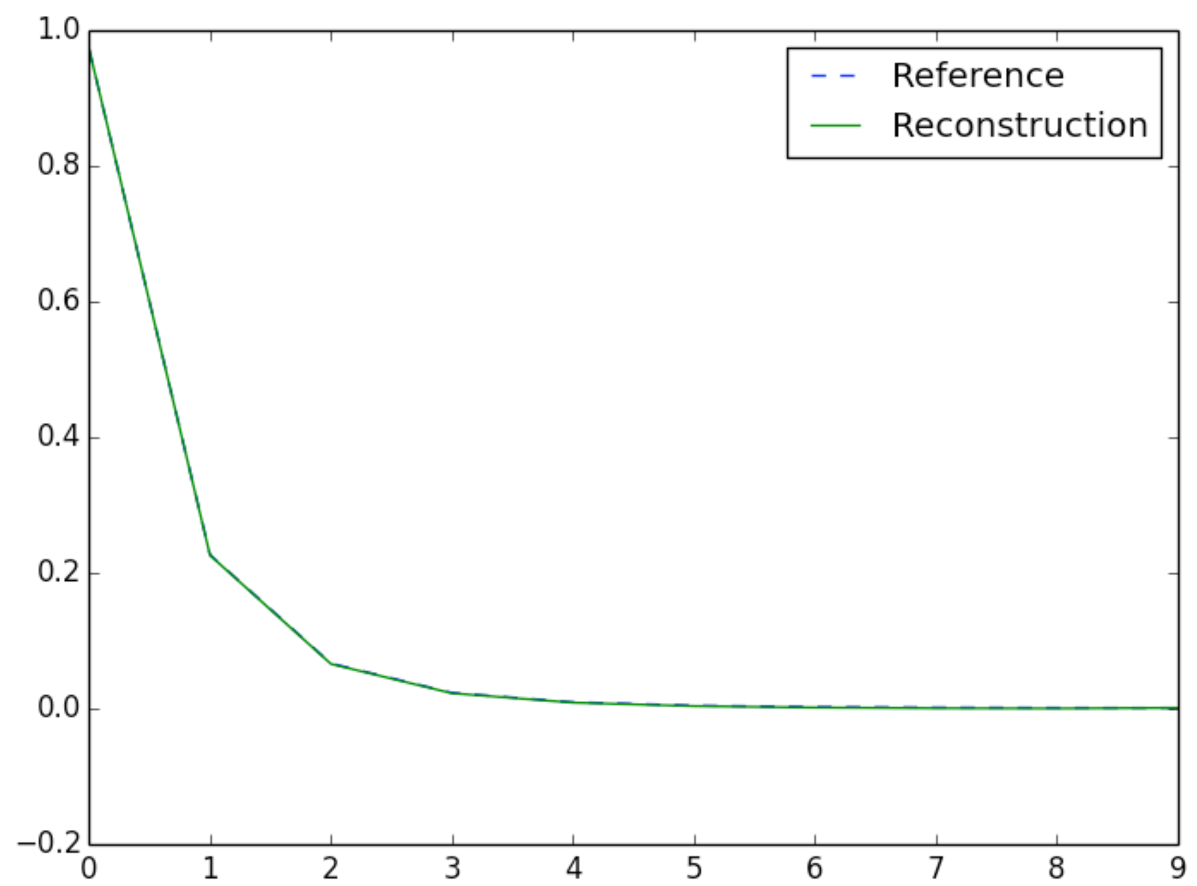
Error



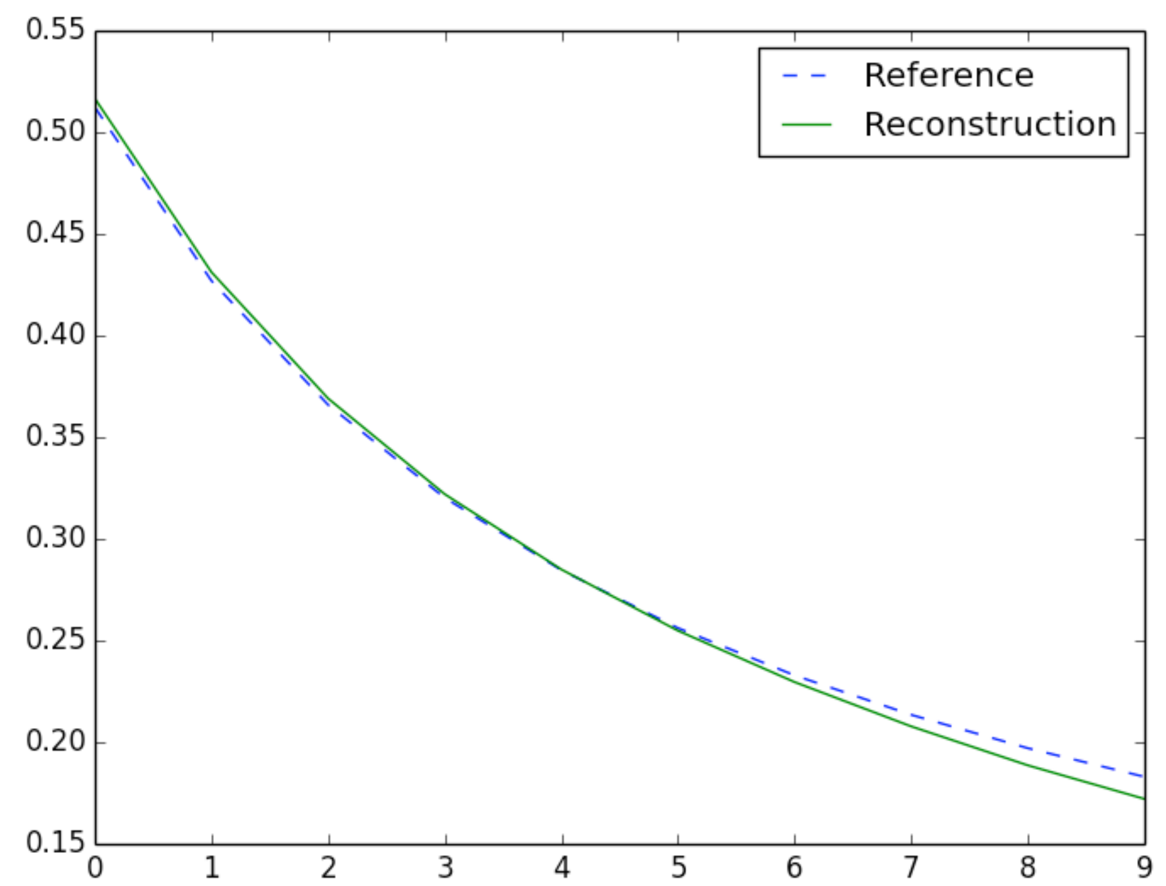


Relative error 0.42% 0.19%

Experiments(Spectra reconstruction)



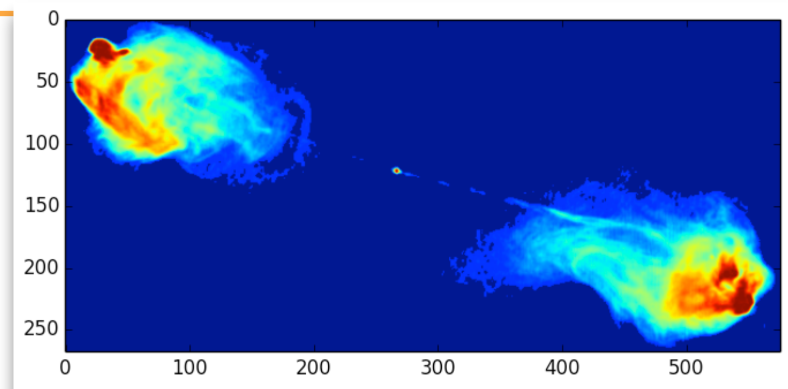
Reconstructed spectrum of S_0 v.s reference

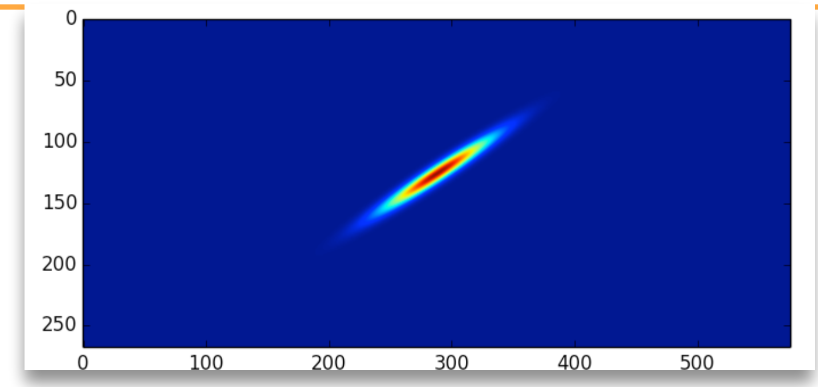


Reconstructed spectrum of S_1 v.s reference

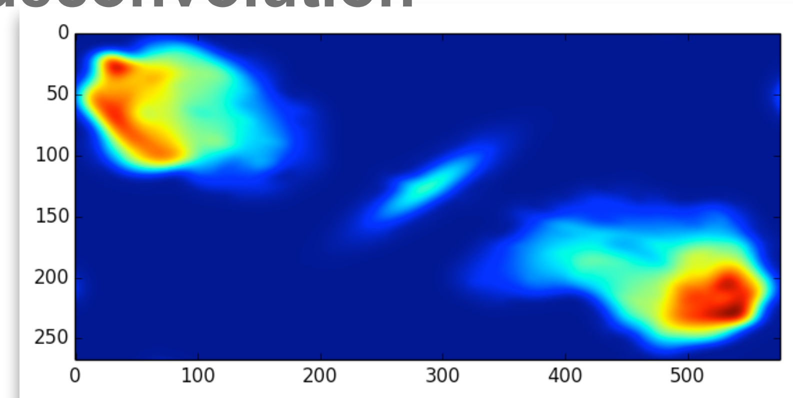
Experiments(Source reconstruction)

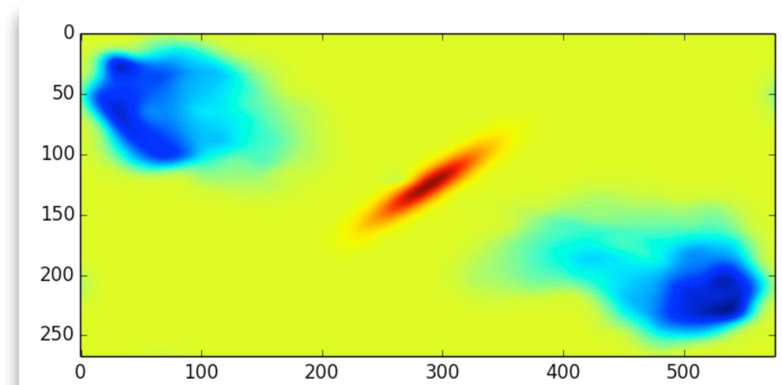
Model sources



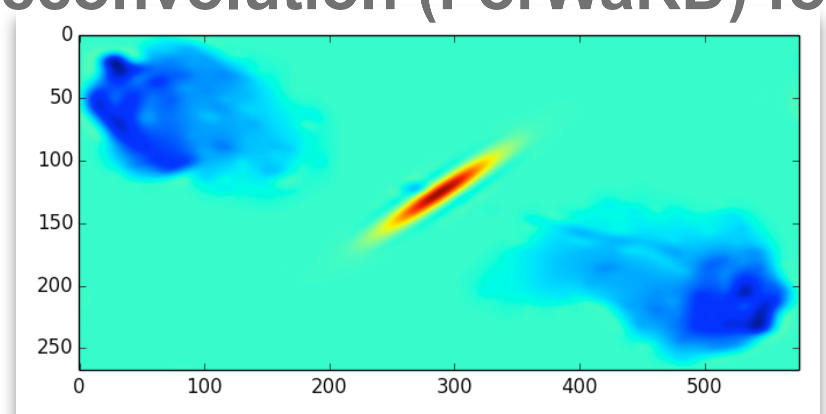


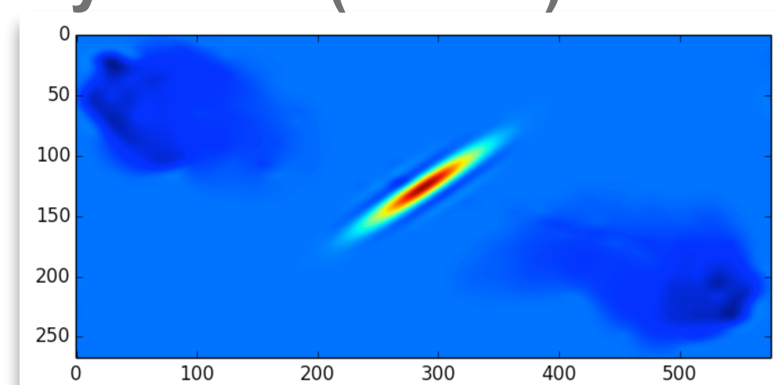
BSS only (GMCA), no deconvolution



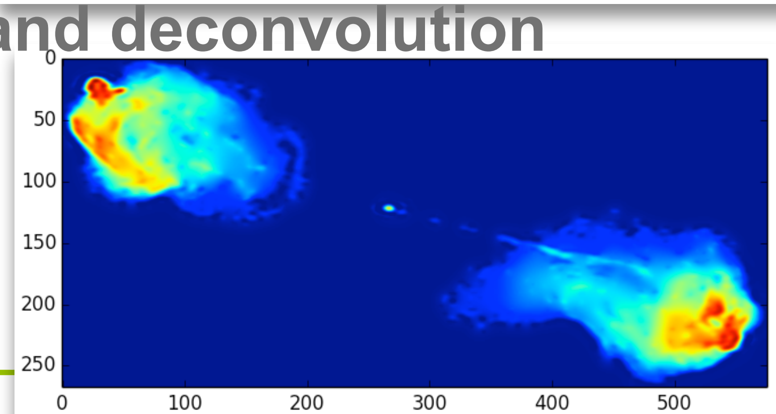


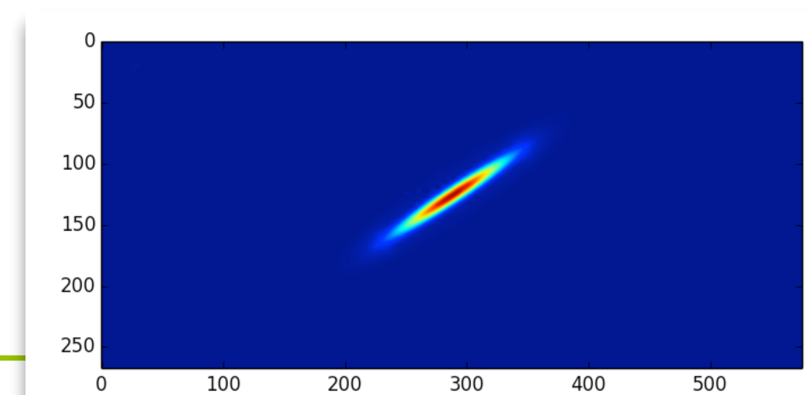
Channel by channel deconvolution (ForWaRD) followed by a BSS (GMCA)





decGMCA : joint BSS and deconvolution

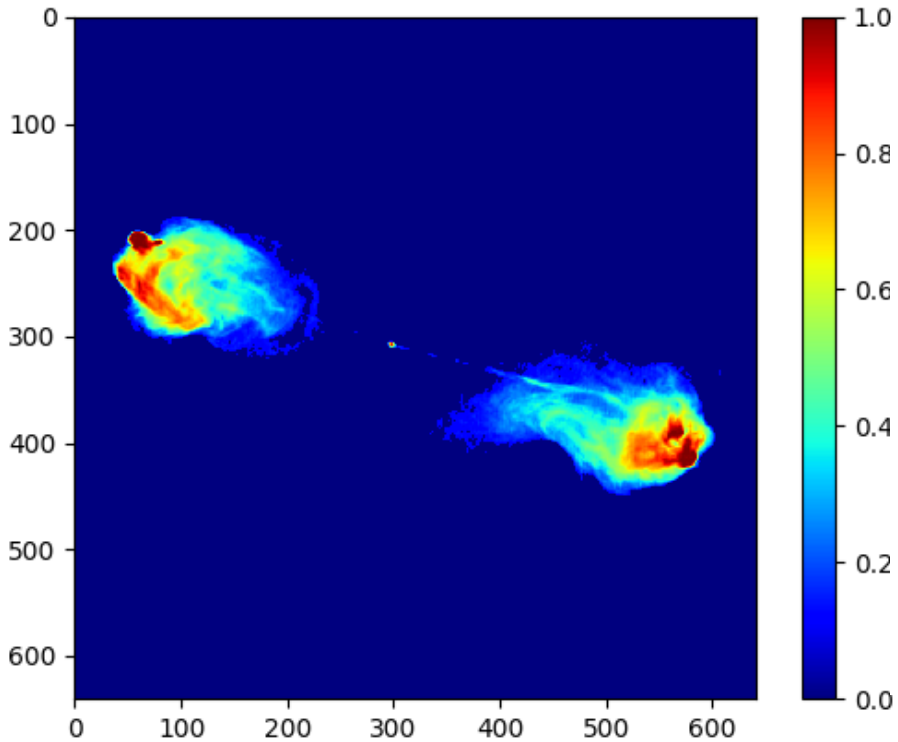


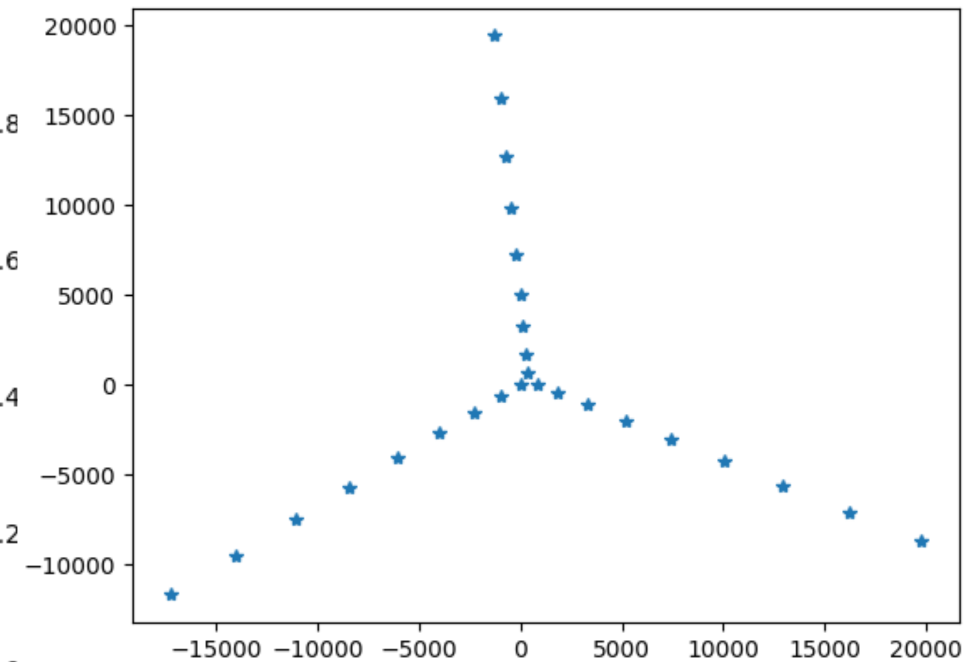


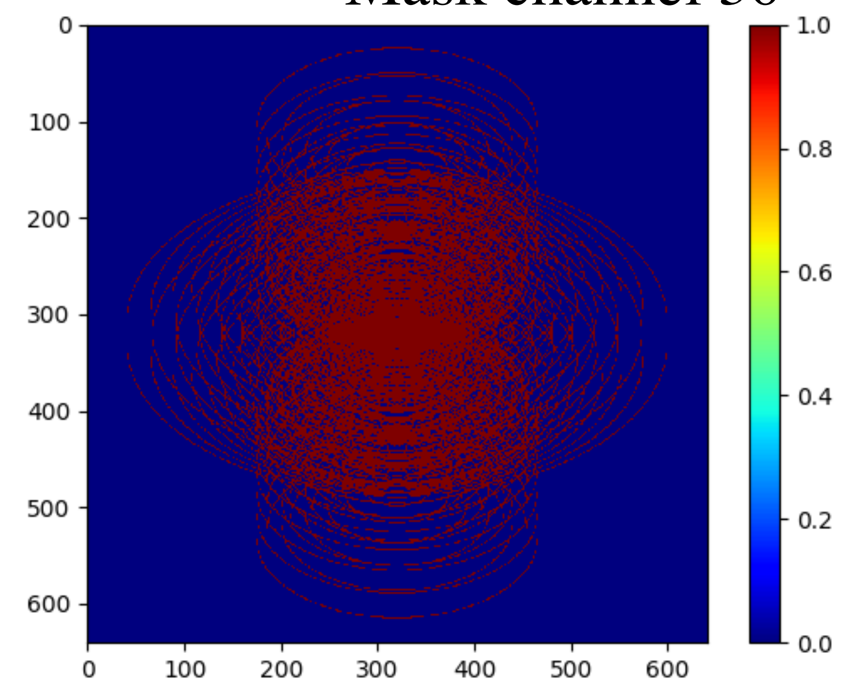
Simulation Cygnus A

- **CygnusA image** (image from NRAO website http://images.nrao.edu/110)

Mask channel 30

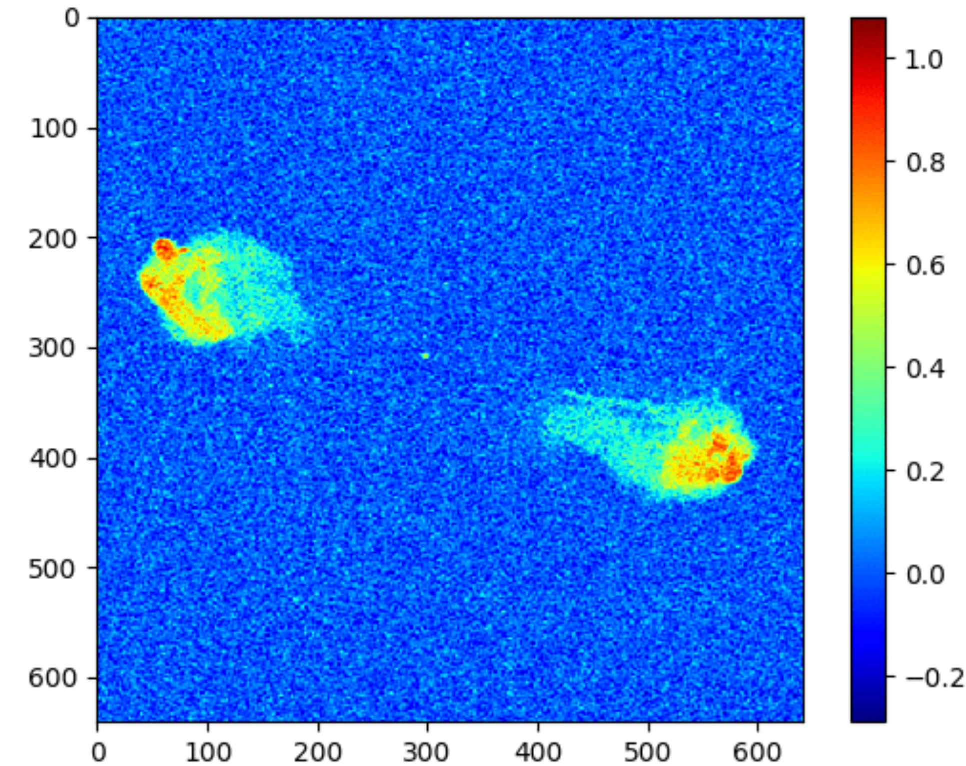






M Cygnus A

VLA

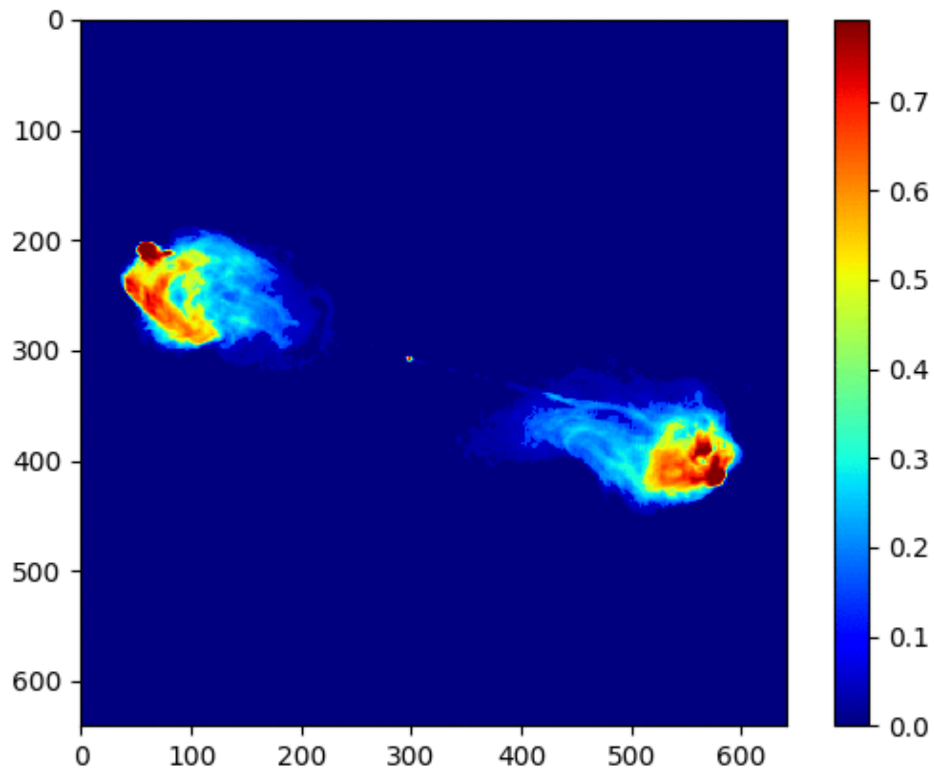


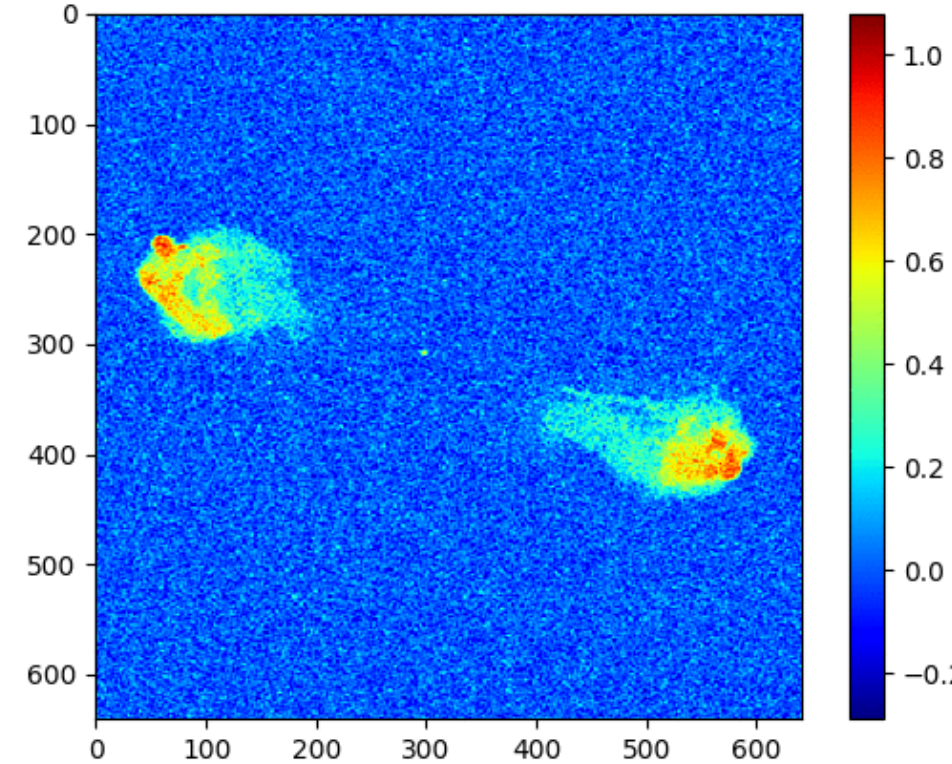
- **Simulation of observation** (VLA setting: 27 antennas)

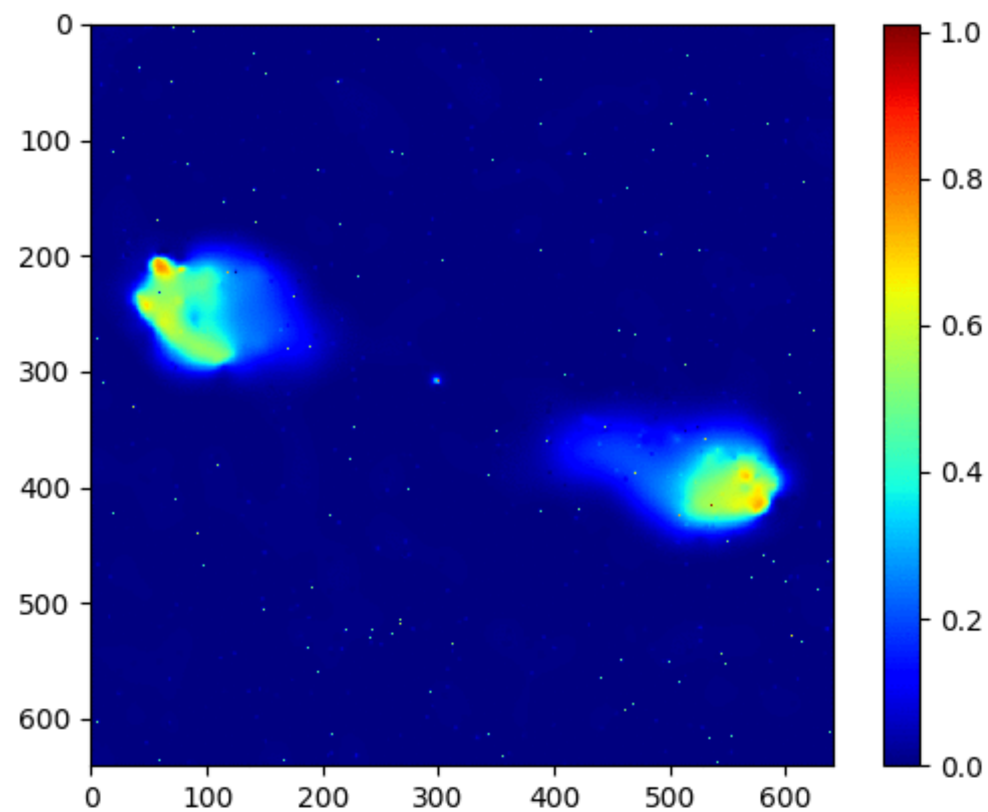
40 channels of observation, interval of 16 MHz between channels

- **Injected noise** (Gaussian noise, σ~0.17)

Simulation Cygnus A







Conclusions

Radio-Imaging

- 2D sparse recovery leads to beautiful results (**resolution**, photometry, etc)
- Extension to 2D-1D: **applied to fast transients** search with good angular resolution.

Hyperspectral image restoration

- Multi/hyperspectral data present channels at different resolutions. A rigorous BSS method should take into account the different channel resolutions.
- decGMCA is an efficient method to solve **jointly the BSS** and the deconvolution problems (**DBSS**).
- It is shown that taking into account joint BSS and deconvolution gives much better results than applying only a BSS or a channel per channel Deconvolution followed by a BSS.

Perspective

- Application of DecGMCA to radio data.
- Application to Weak Lensing and EoR.
- Computation time (HPC).