When Astrophysics meets Medical Imaging

Jean-Luc Starck
http://jstarck.cosmostat.org

CEA, AIM/Service d'Astrophysique, France
Data sampling in the 2D/3D Fourier domain:
Sampling in the 2D Fourier domain

(u,v) plane sampling

RADIO-ASTRONOMY: LOFAR Cygnus A Data

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Key axis for exploiting the future 11.7 Tesla magnet at NeuroSpin (ISEULT)

Compressed Sensing (CS-MRI)

Data acquisition → Random undersampling → Image reconstruction → MR Image

Nonlinear

Sparsity in wavelet basis

Radio-telescope (Squared kilometer Array under construction)
Key axis for exploiting the future 11.7 Tesla magnet at NeuroSpin (ISEULT)

**Stakes in MRI:** Reduce acquisition time
- Improve patient comfort, decrease exam cost
- Avoid heating tissues at high magnetic field
- Limit patient’s movement artifacts
- Achieve very high resolution in space, time, …

**Stakes in Radio-Astronomy**
- Achieve very high resolution in space, time
- …

COSMIC: COMPRESSED SENSING FOR MAGNETIC RESONANCE IMAGING & COSMOLOGY

7T MRI scanner@NeuroSpin

Radio-telescope (Squared kilometer Array under construction)
When Astrophysics meets Medical Imaging

- Part I: Sparsity and Compressed Sensing
- Part II: Data Acquisition and Inverse Problems
- Part III: Perspectives - Hyperspectral Imaging
Abel Prize 2017: Yves Meyer wins 'maths Nobel' for work on wavelets

Frenchman wins prestigious prize for theory that links maths, information technology and computer science

French mathematician Yves Meyer was today awarded the 2017 Abel Prize for his work on wavelets, a mathematical theory with applications in data compression, medical imaging and the detection of gravitational waves.
Paradigm Shift in Statistics/Signal Processing

**20th century**
- Modeling: band limited signals
- Sampling: Shannon Nyquist sampling
- Inverse Problems: linear $l_2$ norm regularization

**21st century**
- Modeling: sparse/compressible signals
- Sampling: Compressed Sensing
- Inverse Problems: non-linear $l_0$-$l_1$ regularization
The top 1% of the coefficients concentrate only 8.66% of the energy. Not sparse...

1% largest coefficients in real space (the others are set to 0)
The wavelet coefficients encode edges and large scale information.

1% largest coefficients in wavelet space (the others are set to 0)

Wavelet transform
1% of the wavelet coefficients concentrate 99.96% of the energy: This can be used as a *prior.*

Reconstruction, after throwing away 99% of the wavelet coefficients
Weak Sparsity or Compressible Signals

Direct Space

Wavelet Space
Weak Sparsity or Compressible Signals

Direct Space

Wavelet Space

Sorted wavelet coefficients

Few large coefficients

Many small coefficients

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What is Sparsity?

A signal $s$ ($n$ samples) can be represented as sum of weighted elements of a given dictionary

$$\Phi = \{\phi_1, \ldots, \phi_K\}$$

**Atoms**

$$S = \sum_{k=1}^{K} \alpha_k \phi_k = \Phi \alpha$$

**Dictionary**

(basis, frame)

Ex: Haar wavelet

Fast calculation of the coefficients

- Analyze the signal through the statistical properties of the coefficients
- Approximation theory uses the sparsity of the coefficients

$$f(x) = c_0 + \sum_{j=0}^{\infty} \sum_{k=0}^{2^j-1} c_{j,k} \psi_{j,k}(x).$$
Inverse Problems: Sparse Recovery

\[ Y = HX + N \]

\[ \min_X \| Y - HX \|^2 + C(X) \]
Inverse Problems: Sparse Recovery

\[ Y = HX + N \]

\[
\min_X \|Y - HX\|^2 + C(X)
\]

Sparse model: \( X = \Phi \alpha \)

\[
\min_{\alpha} \|Y - H\Phi \alpha\|^2 + \lambda \|\alpha\|_p^p
\]
Inverse Problems: Sparse Recovery

\[ Y = HX + N \]

\[
\min_X \|Y - HX\|^2 + C(X)
\]

Sparse model: \[ X = \Phi\alpha \]

\[
\min_\alpha \|Y - H\Phi\alpha\|^2 + \lambda \|\alpha\|_p^p
\]

Optimization (proximal theory)
Inverse Problems: Sparse Recovery

\[ Y = HX + N \]

\[
\min_X \| Y - HX \|^2 + C(X)
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Sparse model: \( X = \Phi \alpha \)

\[
\min_\alpha \| Y - H\Phi \alpha \|^2 + \lambda \| \alpha \|_p^p
\]

Optimization
(proximal theory)

Data Representation
(harmonic analysis, machine learning)
Inverse Problems: Sparse Recovery

\[ Y = HX + N \]

\[
\min_X \|Y - HX\|^2 + C(X)
\]

Sparse model: \( X = \Phi \alpha \)

\[
\min_\alpha \|Y - H\Phi \alpha\|^2 + \lambda \|\alpha\|_p^p
\]

- **Optimization** (proximal theory)
- **Data Representation** (harmonic analysis, machine learning)
- **Noise Modeling** (Gaussian, Poisson, etc)

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$\|X\|_p = \left( \sum_i |X_i|^p \right)^{\frac{1}{p}}$

$p < 2$
Compressed Sensing

A non linear sampling theorem

“Signals with exactly \( K \) components different from zero can be recovered perfectly from \( \sim K \log N \) incoherent measurements”

Replace samples with few linear projections \( Y = HX \)

Incoherent Measurements

\[
Y \quad \overset{H}{\longrightarrow} \quad X = K \text{ sparse signal}
\]

Measurement System

\( M \times N \)

\( \begin{align*}
M \times 1 \\
K \text{ nonzero entries}
\end{align*} \)

\( K < M << N \)

Reconstruction via non linear processing:

\[
\min_X \| X \|_1 \quad \text{s.t.} \quad Y = HX
\]
Mutual Incoherence

In practice, \( X \) is sparse in a dictionary: \( X = \Phi \alpha \)

optimally incoherent

ex: Fourier/Dirac

perfectly coherent ensembles

Incoherence between a sparse representation and a measurement matrix (i.e. mutual incoherence):

\[
\mu(H, \Phi) = \max_{i,j} |\langle H_i, \phi_j \rangle| \]

measures how an atom of the sparse representation spreads in the measurement ensemble.

\[
\frac{1}{\sqrt{n}} \leq \mu(H, \Phi) \leq 1
\]
In the case of exactly $K$-sparse signals, and perfect measurements (no noise), the following result holds:

If  \[ m \geq C \mu(H, \Phi)^2 K \log(n) \]

Then  \[ \min_{\alpha} \|\alpha\|_{1} \text{ s.t. } y = H\Phi\alpha \]

Provides a perfect reconstruction
Effective Sparse Recovery algorithms

CS recovery requires solves convex but non-smooth minimization problems

$$\min_{\alpha} \| Y - H\Phi\alpha \|^2 + J(\alpha)$$

sparsity penalization in $\Phi$

Since a decade, proximal algorithms can solve efficiently these problems, even in large-scale settings:

$$X^{(n+1)} = \text{prox}_{\gamma J} \left( X^{(n)} + \gamma H^T(Y - HX^{(n)}) \right)$$

Proximal operator

Gradient descent step

Ex. Forward splitting Algorithm (Combettes et al. 2005), FISTA (Beck & Teboulle 2009), etc.
Sparse Recovery & Inverse Problems

\[ Y = HX + N \]

\[ X = \Phi \alpha \]

and \( \alpha \) is **sparse** or **compressible**

\[
\min_{\alpha} \| Y - H\Phi \alpha \|^2 + \lambda \| \alpha \|^p
\]

- Denoising
- Deconvolution
- Component Separation
- **Inpainting**
- Blind Source Separation
- Minimization algorithms
- Compressed Sensing
Interpolation of Missing Data

Inpainting


- Period detection in temporal series

- Bad pixels, cosmic rays, point sources in 2D images, ...
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\[ Y = HX + N \quad X = \Phi \alpha \quad \text{and} \quad \alpha \quad \text{is sparse or compressible} \]

\[ \mu(H, \Phi) = \max_{i,j} |\langle H_i, \phi_j \rangle| \]

\[ \min_{\alpha} \|Y - H\Phi \alpha\|^2 + \lambda \|\alpha\|_p^p \]

\[ y \quad H \quad \Phi \quad \alpha \]

\[ \text{Measurement System} \]

\[ |\alpha| \quad K \quad \text{sorted index} \quad N \]

\[ \mu \text{power-law decay} \]
Sparse Representations

- **Local DCT**
  - Stationary textures
  - Locally oscillatory

- **Wavelet transform**
  - Piecewise smooth
  - Isotropic structures

- **Curvelet transform**
  - Piecewise smooth, edge

- **Dictionary Learning**
The STARLET Transform

Sorted wavelet coefficients

Few large coefficients

Many small coefficients

x is approximately sparse in $\Phi$

Starlet transform
(isotropic undecimated wavelet transform)
Compressed Sensing & LOFAR Cygnus A Data


http://arxiv.org/abs/1406.7242
Colors: reconstructed 512x512 image of Cygnus A at 151 MHz (with resolution 2.8" and a pixel size of 1"). Contours levels are [1,2,3,4,5,6,9,13,17,21,25,30,35,37,40] Jy/Beam from a 327.5 MHz Cyg A VLA image (Project AK570) at 2.5" angular resolution and a pixel size of 0.5". **Recovered features in the CS image correspond to real structures observed at higher frequencies.**
The Acquisition Matrix

\[ Y = HX + N \quad X = \Phi \alpha \quad \text{and} \quad \alpha \text{ is sparse or compressible} \]

\[ \mu(H, \Phi) = \max_{i,j} |\langle H_i, \phi_j \rangle| \]

\[ \min_{\alpha} \|Y - H\Phi \alpha\|^2 + \lambda \|\alpha\|_p^p \]

Measurement System
IBIS Camera on board of the Integral satellite

Launch, October 4, 2002.
Gamma Ray Instruments (Integral) - Acquisition with coded masks

INTEGRAL/IBIS Coded Mask

Crab Nebula Integral Observation

Courtesy I. Caballero, J. Rodriguez (AIM/Saclay)
Single Pixel Camera (Rice University)
http://dsp.rice.edu/cscamera
SVOM (future French-Chinese Gamma-Ray Burst mission)

- ECLAIRs france-chinese satellite ‘SVOM’
  Gamma-ray detection in energy range 4 - 120 keV
  Coded mask imaging (at 460 mm of the detector plane)

Physical mask pattern
(46 x 46 pixels of 11.7 mm)
Design of new sampling patterns (k-space trajectories)


=> See Carole’s Poster
Sequence parameters:
- N=512
- FOV=200x200 mm²
- TR=550 ms and TE=30 ms
- α=25°
- BW=32.55 Hz/px
- Tobs=30.72 ms
- Slice thickness: 3 mm
- Single channel receiver coil
- Ex-vivo baboon brain

λ=2.10⁻⁵
H2020 HyperSPACE submitted project for CS remote sensing
Hyperspectral Data

Ground Truth

Mixtures

\[ X = AE \]

PSF \( H \)

Data

\[ Y = HX + N \]
Hyperspectral Sparsity

\[ Y_i = H_i \ast X_i + N \]

\[ X_i = \sum_{s=1}^{S} a_{i,s} E_s \quad E_s = \Phi \alpha_s \]

\[ Y_i = H_i \ast \sum_{s=1}^{S} a_{i,s} \Phi \alpha_s + N \]

Joint Blind Source Separation and Deconvolution Problem

\[ \min_{A,\alpha_s} \| Y_i - H_i \ast \sum_{s=1}^{S} a_{i,s} \Phi \alpha_s \|^2 \]

\[ \Rightarrow \text{See Ming’s Poster} \]

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Conclusions

✓ MRI and Radio-Astronomy Imaging present similar inverse problems
  ➡ Sparse Recovery is very efficient
  ➡ Clear link with Compressed Sensing Theory

✓ Radio-Astronomy
  ➡ Improve the resolution by a factor 2.

✓ MRI Imaging
  ➡ Divide the acquisition time by a factor 8.

✓ Perspective: CS Hyperspectral Imaging
  ➡ Big computational challenges
  ➡ New sparse modeling
  ➡ New applications (agriculture, medical, etc)