When Astrophysics meets Medical Imaging

Jean-Luc Starck

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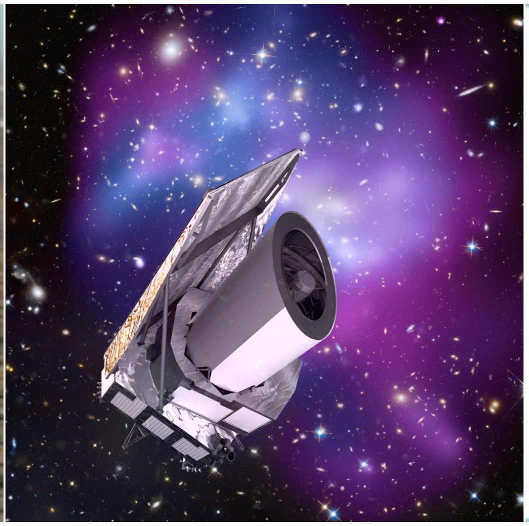
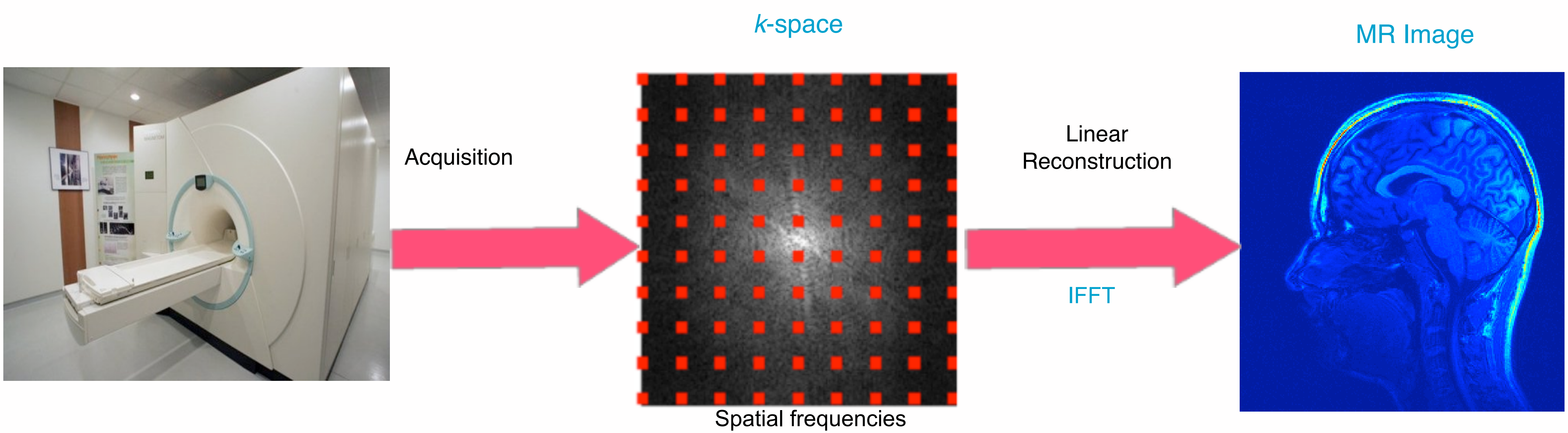


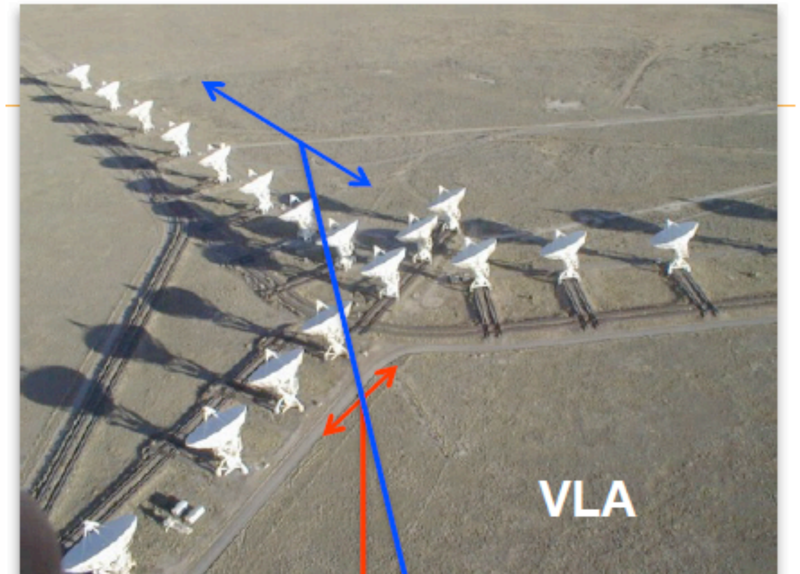
COMMONALITIES BETWEEN MRI & RADIO-INTERFEROMETRY

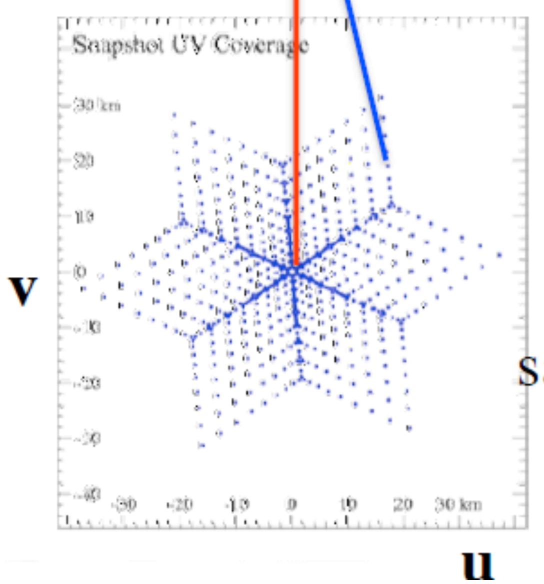
- Data sampling in the 2D/3D Fourier domain:



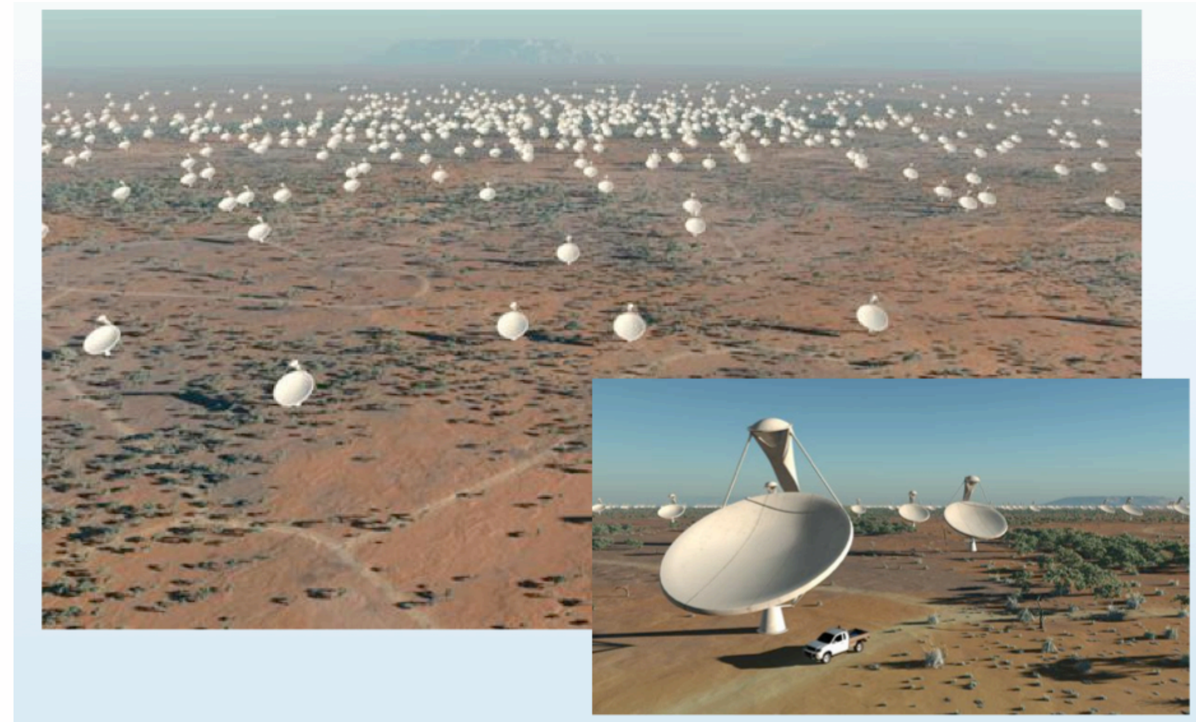
7Tesla MRI Scanner @NeuroSpin

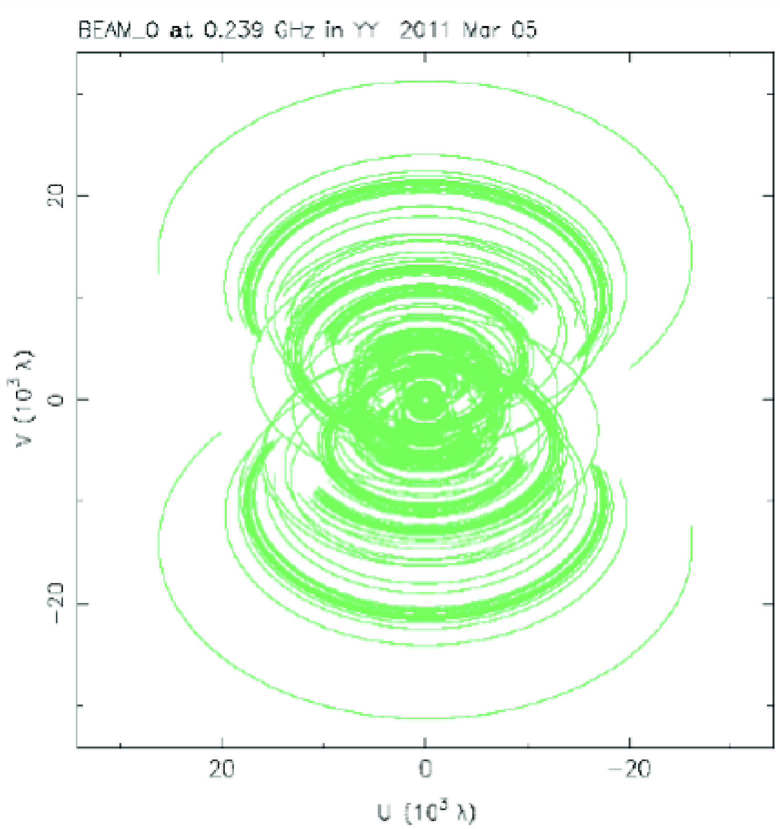
RADIO-ASTRONOMY : LOFAR Cygnus A Data

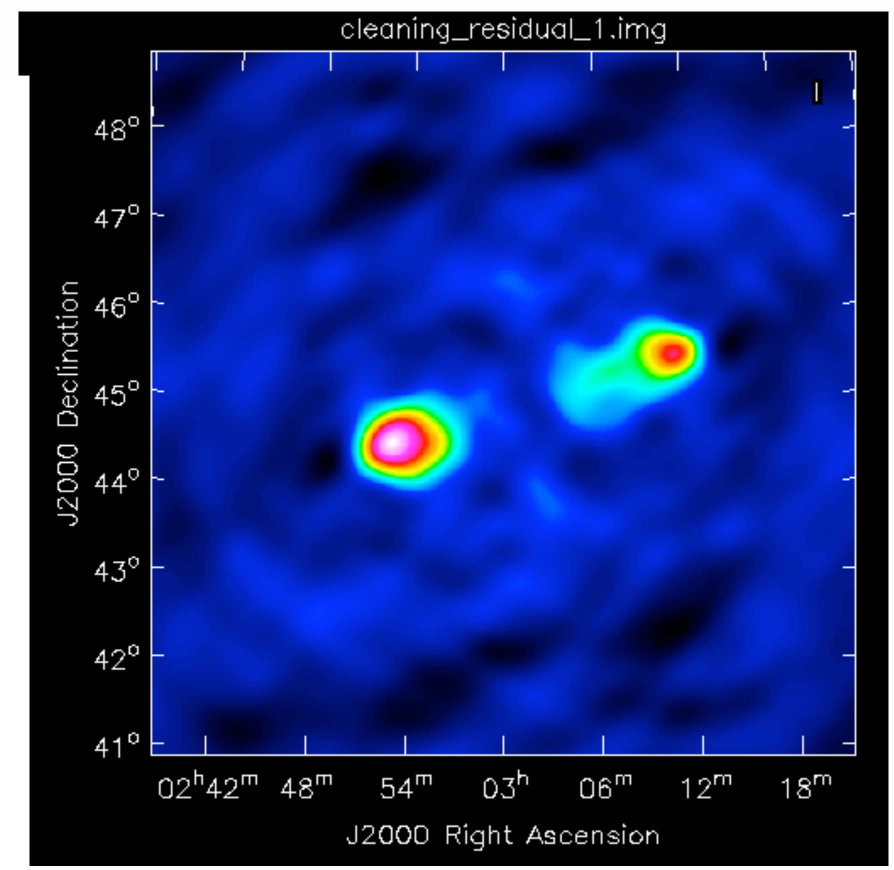




(u,v)
plane
sampling







CosmoStat Lab

Sampling in the 2D Fourier domain

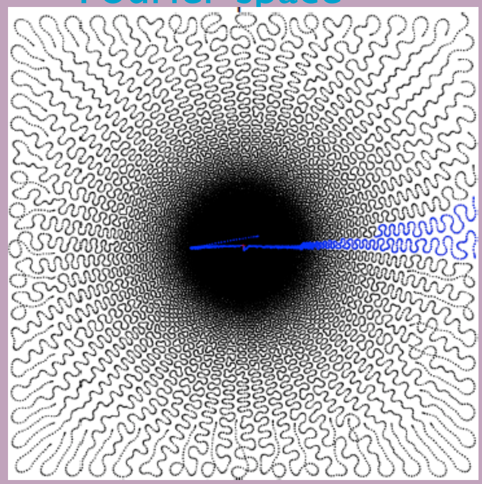
- Key axis for exploiting the future 11.7 Tesla magnet at NeuroSpin (ISEULT)

7T MRI scanner@NeuroSpin

Data acquisition

Compressed Sensing (CS-MRI)

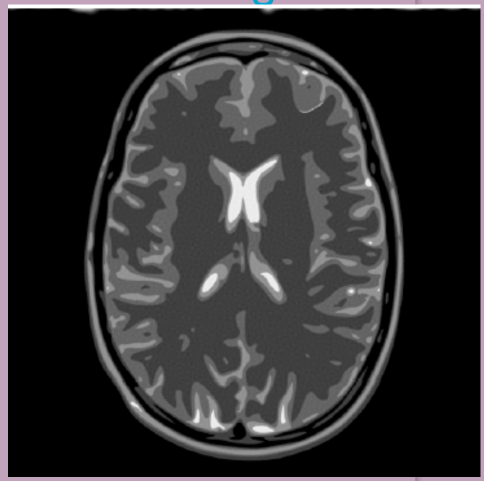
Fourier space



Random undersampling

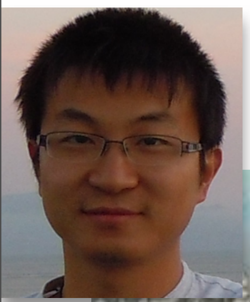
Nonlinear Image reconstruction

MR Image



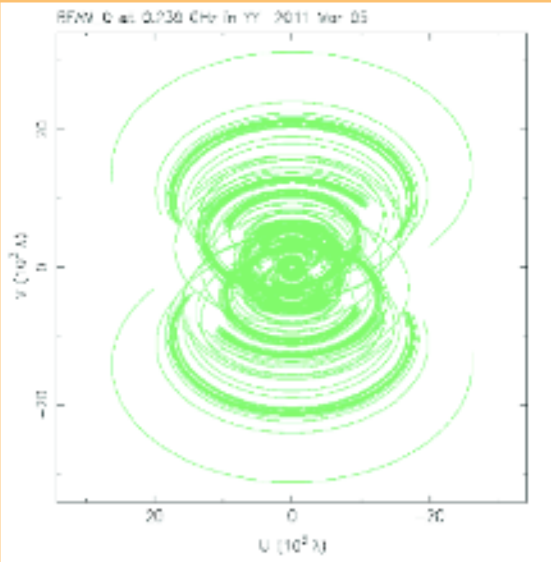
Sparsity in wavelet basis

- Radio-telescope (Squared kilometer Array under construction)

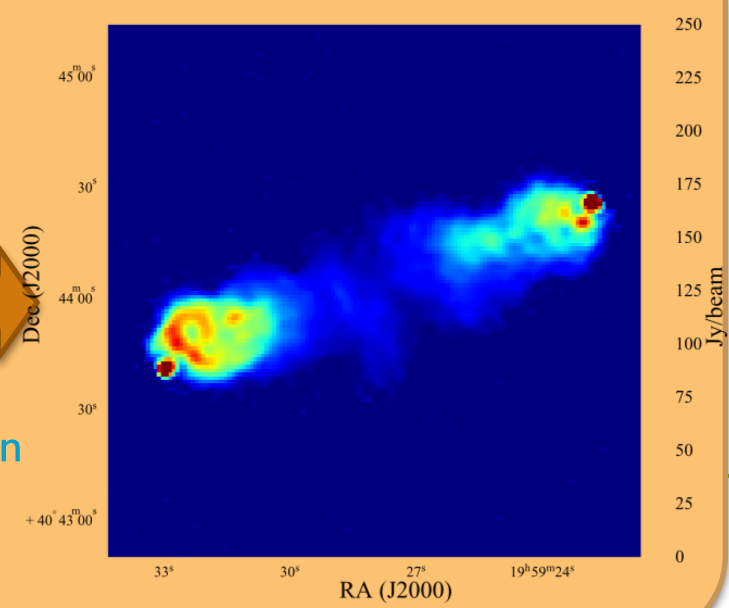


Data acquisition

Fourier space



Nonlinear Image reconstruction

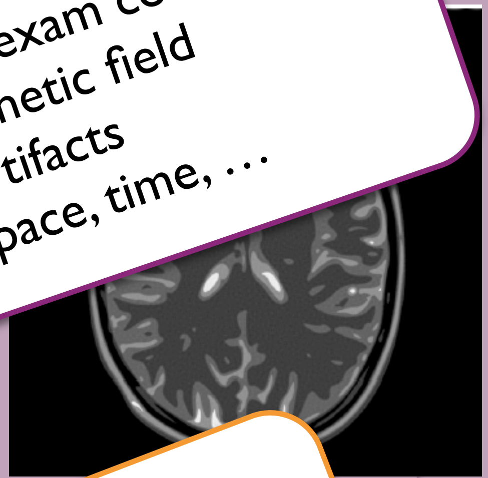


- Key axis for exploiting the future 11.7 Tesla magnet at NeuroSpin

7T MRI scanner@NeuroSpin

Data acquisition

Com



ction

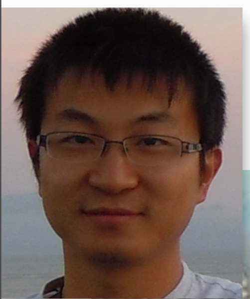
ndersampling

basis

Stakes in MRI: **Reduce acquisition time**

- Improve patient comfort, decrease exam cost
- Avoid heating tissues at high magnetic field
- Limit patient's movement artifacts
- Achieve very high resolution in space, time, …

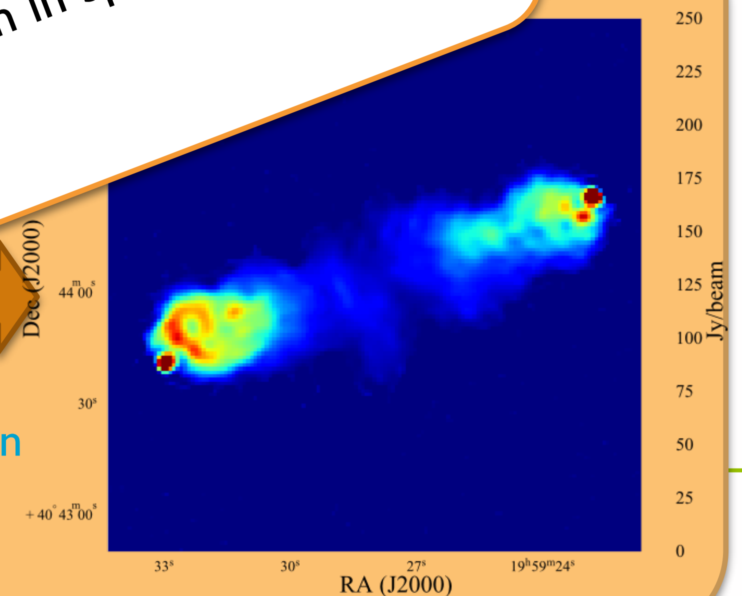
- Radio-telescope (Squared kilometer Array under



Data acquisition

Fourie

Image reconstruction



Stakes in Radio-Astronomy

- Achieve very high resolution in space, time
- …

When Astrophysics meets Medical Imaging

- Part I: Sparsity and Compressed Sensing

- Part II: Data Acquisition and Inverse Problems

- Part III: Perspectives - Hyperspectral Imaging theguardian

Abel Prize 2017: Yves Meyer wins 'maths Nobel' for work on wavelets

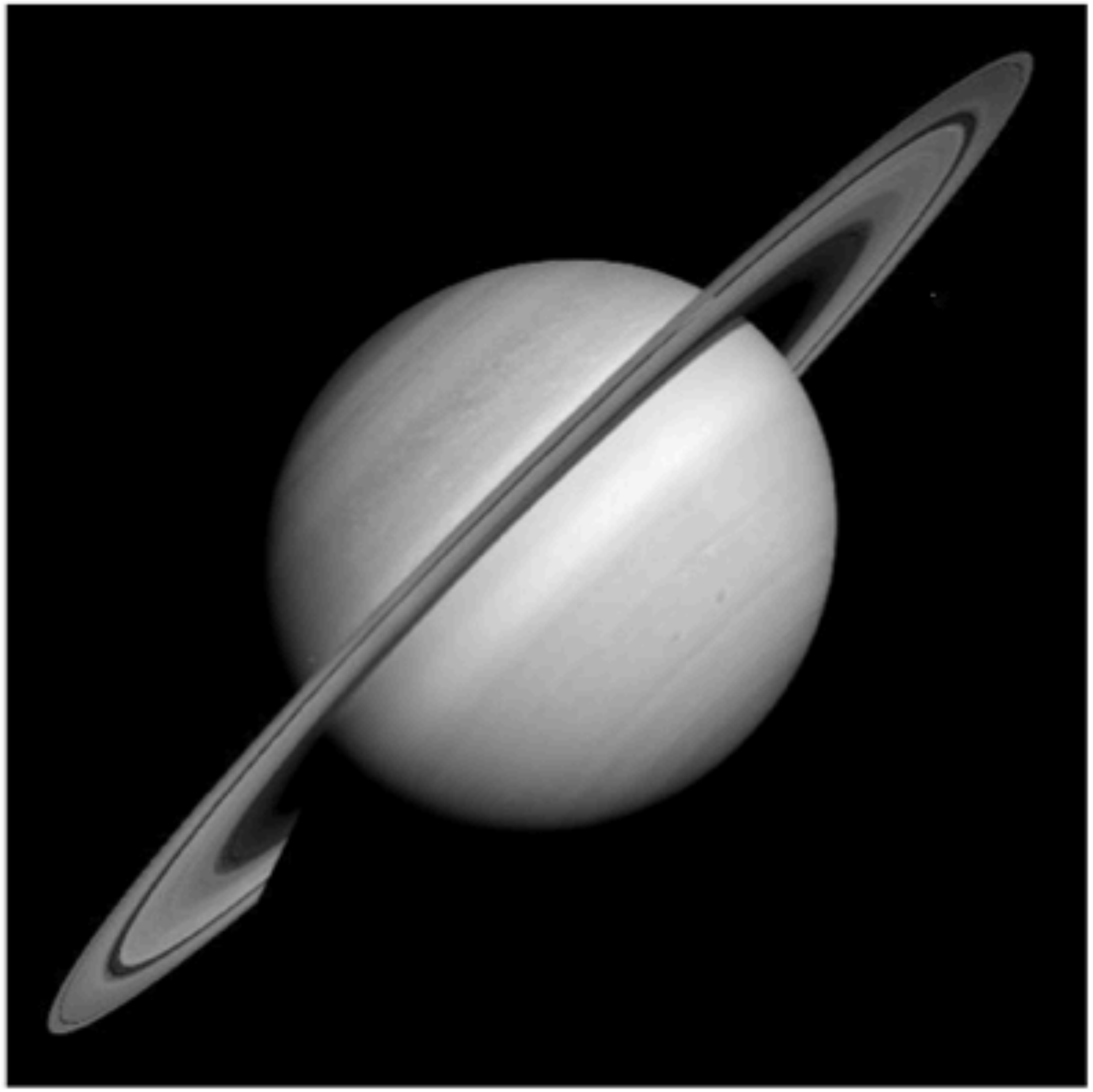
Frenchman wins prestigious prize for theory that links maths, information technology and computer science

Yves Meyer Photograph: B. Eymann/Academie des sciences

French mathematician Yves Meyer was today awarded the 2017 Abel Prize for his work on wavelets, a mathematical theory with applications in data compression, medical imaging and the detection of gravitational waves.

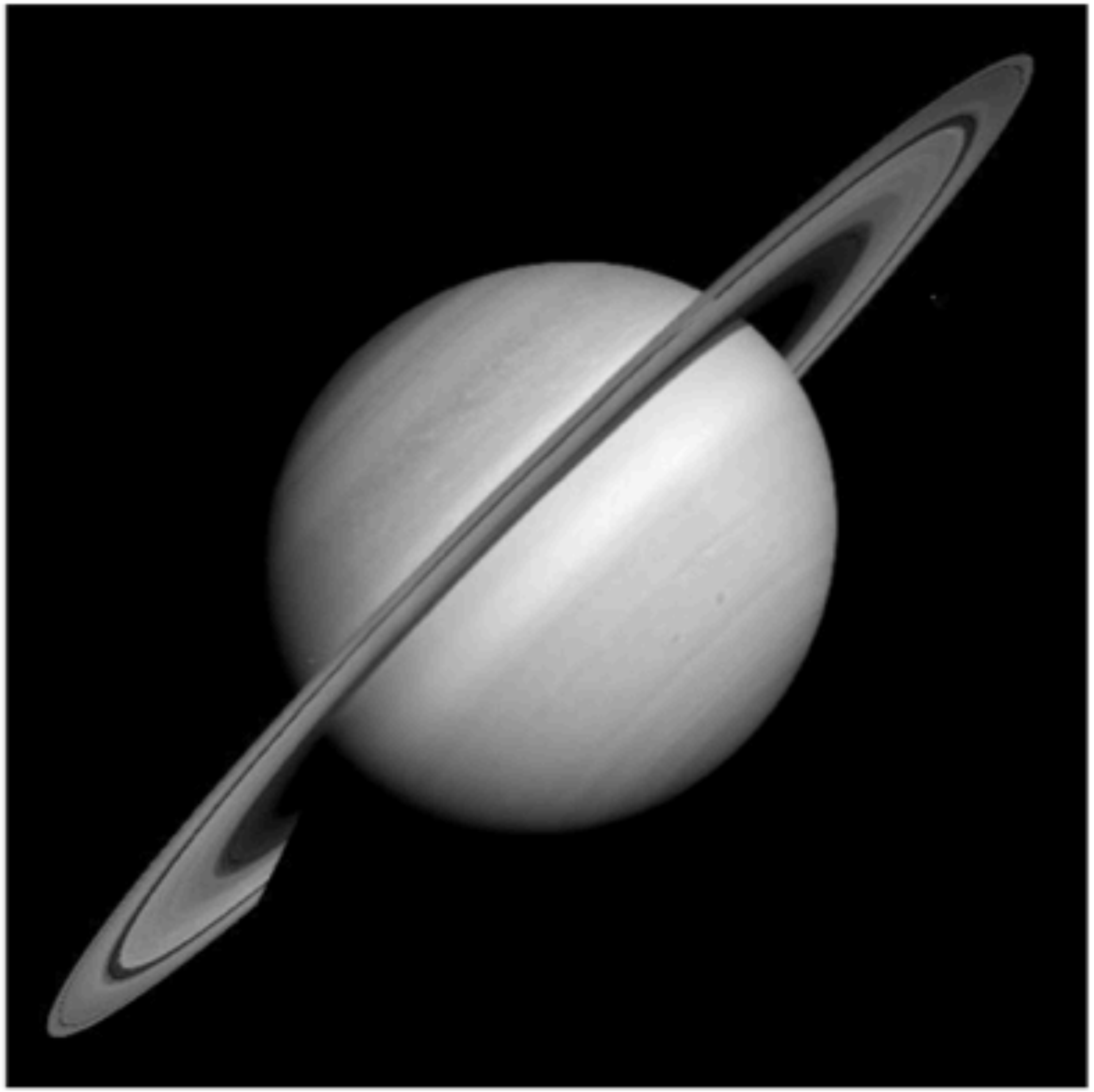
Paradigm Shift in Statistics/Signal Processing

	Modeling	Sampling	Inverse Problems
20th century	band limited signals	Shannon Nyquist sampling	linear l2 norm regularization
↓	↓	↓	↓
21st century	sparse/compressible signals	Compressed Sensing	non-linear l0-l1 regularization



1% largest coefficients in real space
(the others are set to 0)

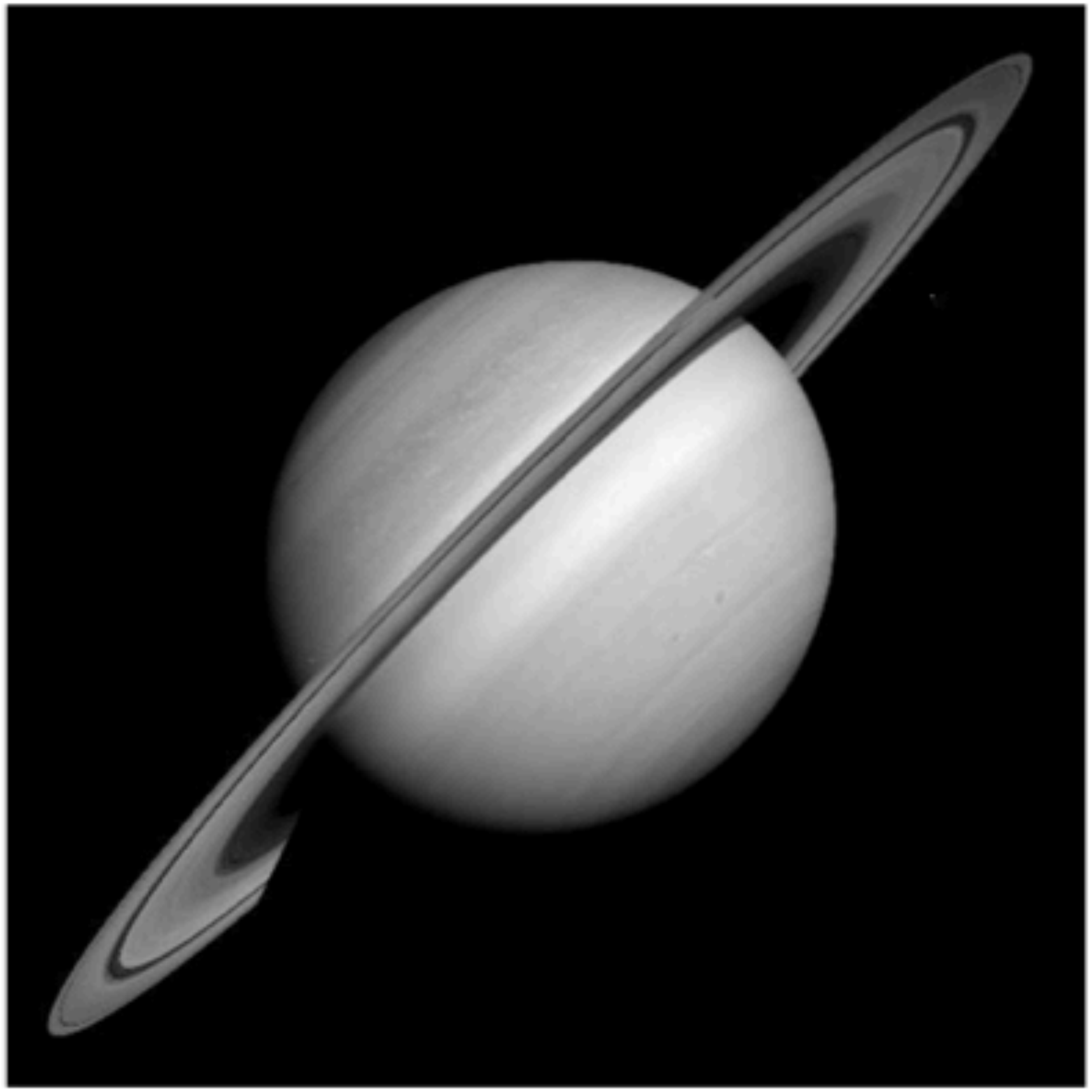
The top 1% of the coefficients concentrate only 8.66% of the energy. Not sparse...

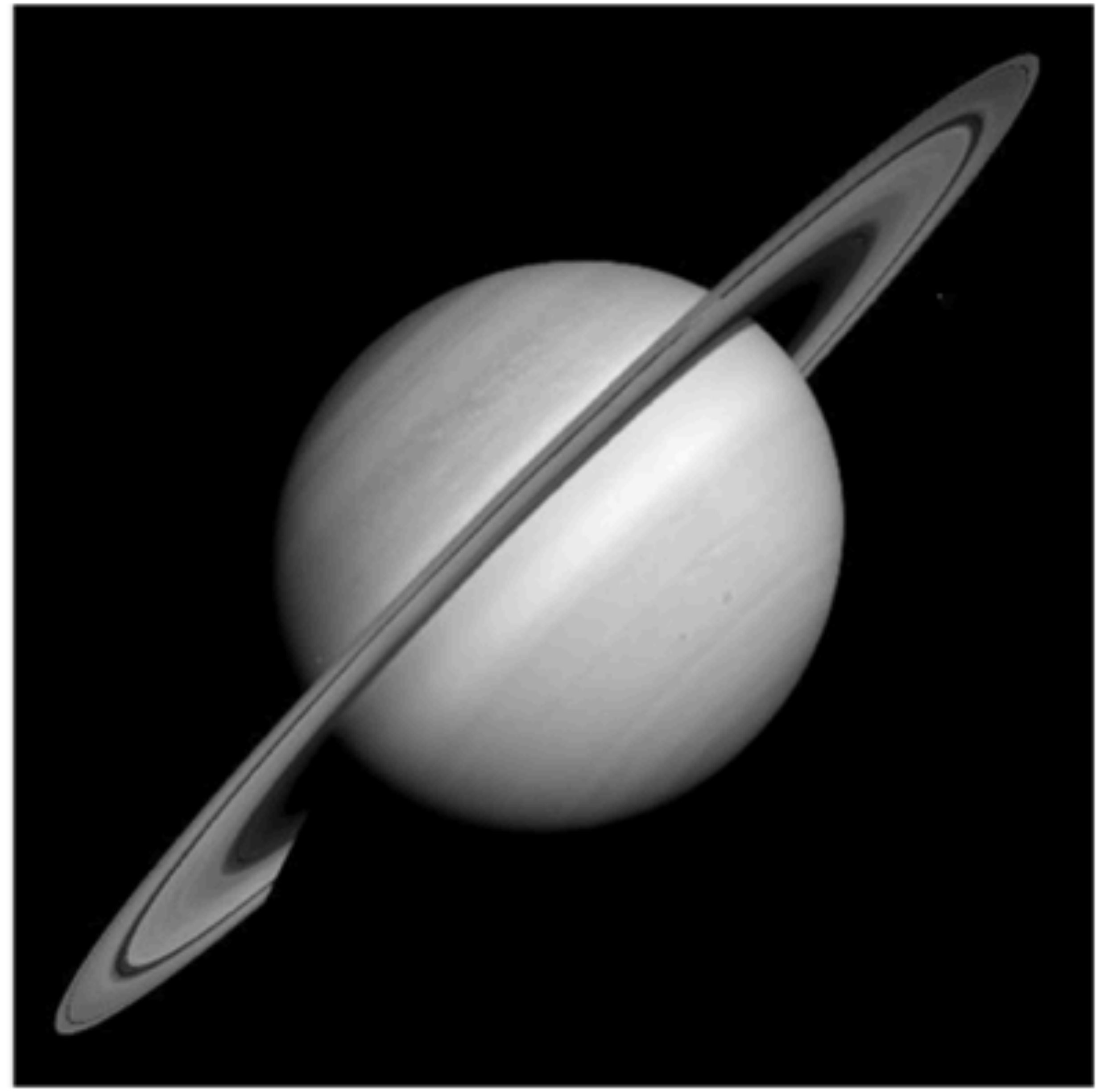


1% largest coefficients in wavelet space
(the others are set to 0)

The wavelet coefficients encode edges and large scale information.

Wavelet transform

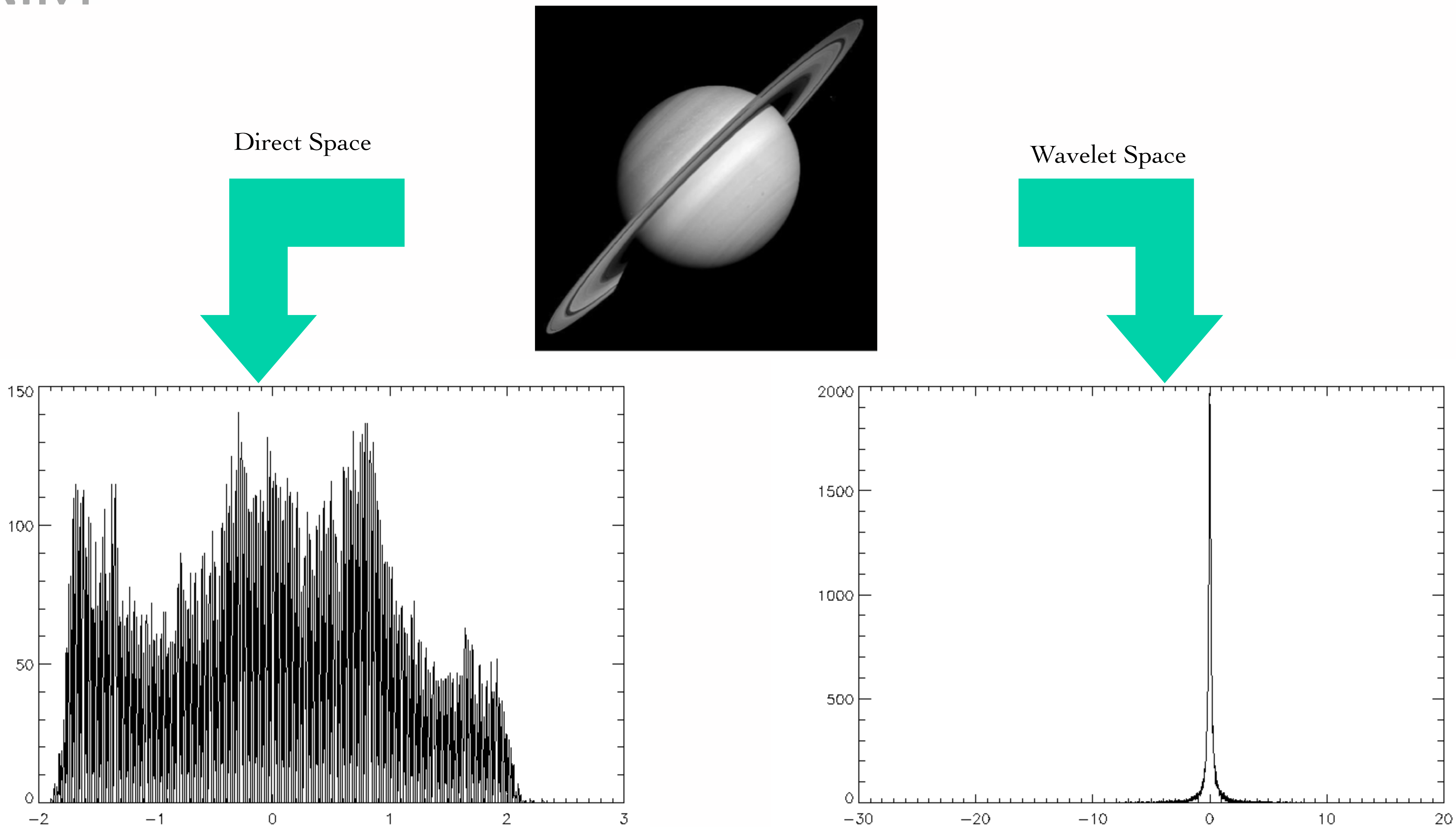




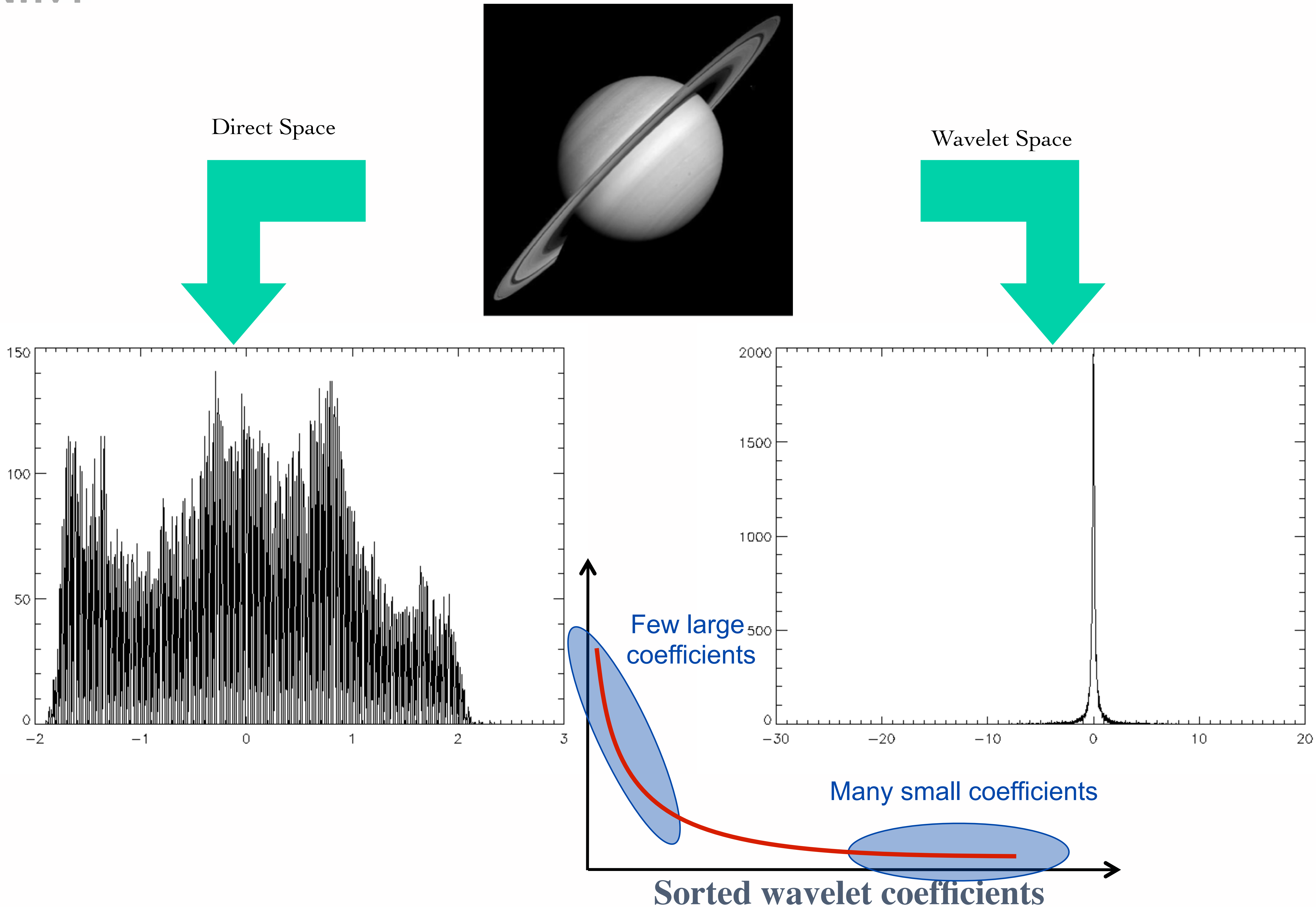
1% of the wavelet coefficients concentrate 99.96% of the energy: This can be used as a *prior.*

Reconstruction, after throwing away 99% of the wavelet coefficients

Weak Sparsity or Compressible Signals



Weak Sparsity or Compressible Signals



What is Sparsity?

A signal *s* (*n* samples) can be represented as sum of weighted elements of a given dictionary

Dictionary (basis, frame)

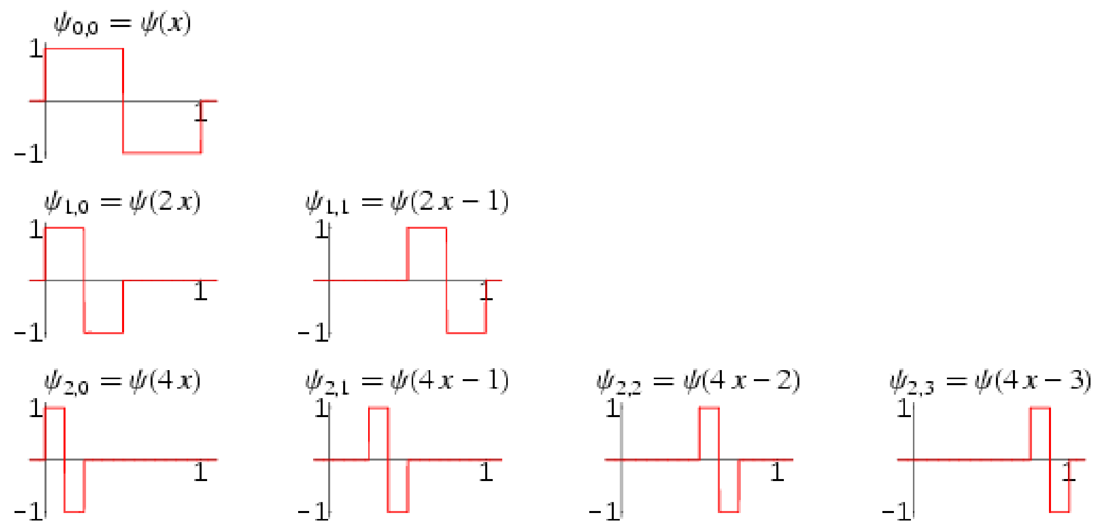
$$\Phi = \{\phi_1, \ldots, \phi_K\}$$

Atoms

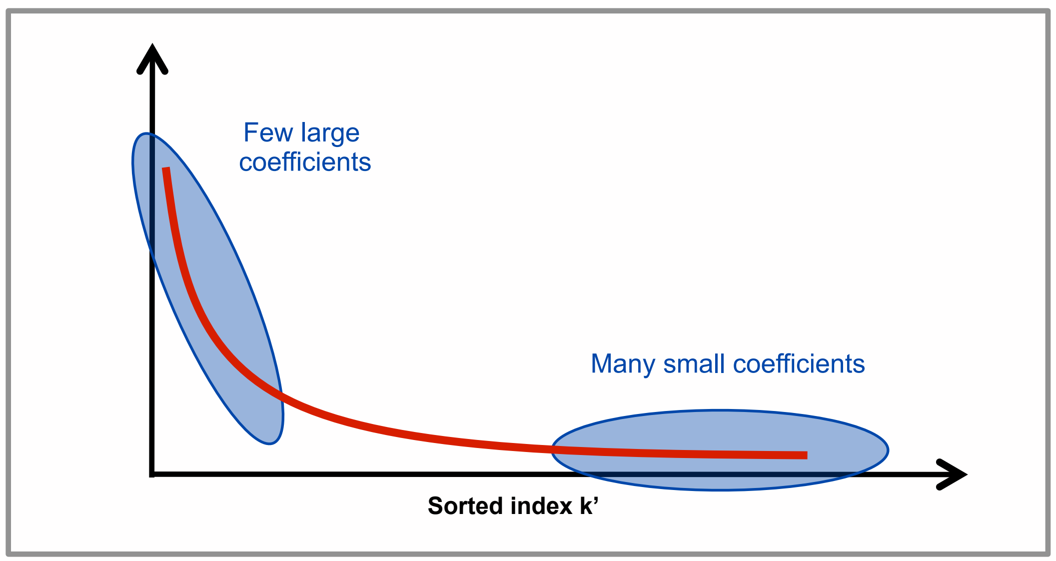
$$s = \sum_{k=1}^{K} \alpha_k \phi_k = \Phi\alpha$$

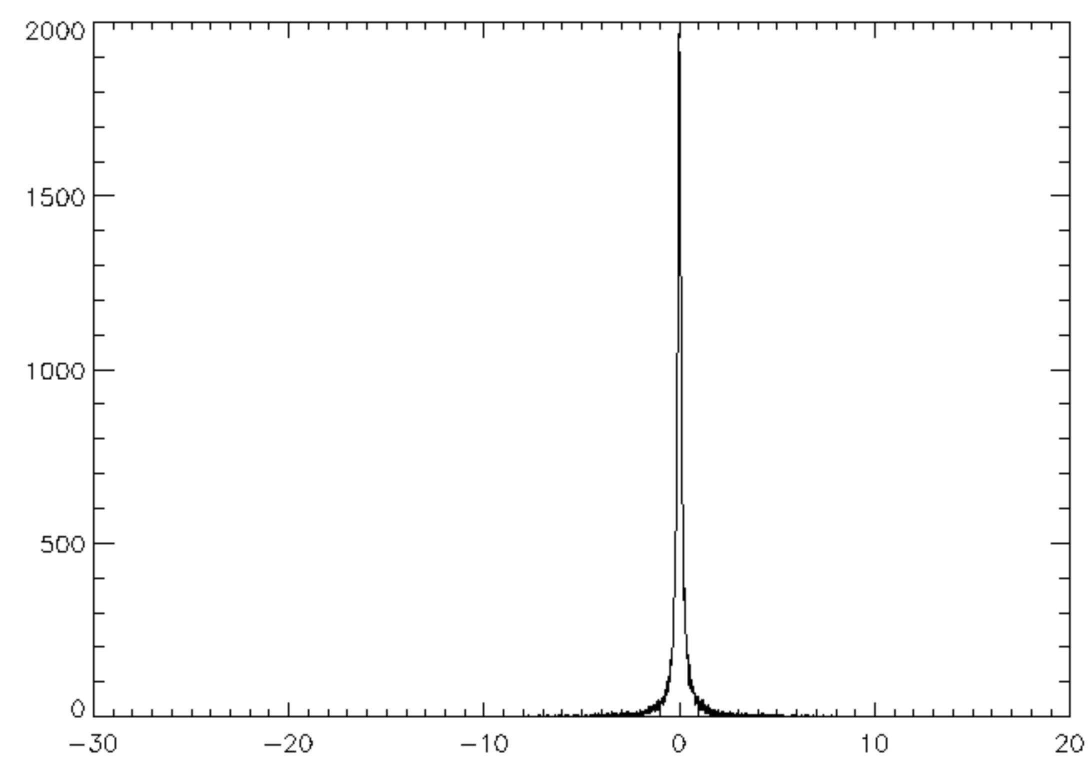
coefficients

Ex: Haar wavelet



$$f(x) = c_0 + \sum_{j=0}^{\infty} \sum_{k=0}^{2^j-1} c_{jk}\, \psi_{jk}(x).$$





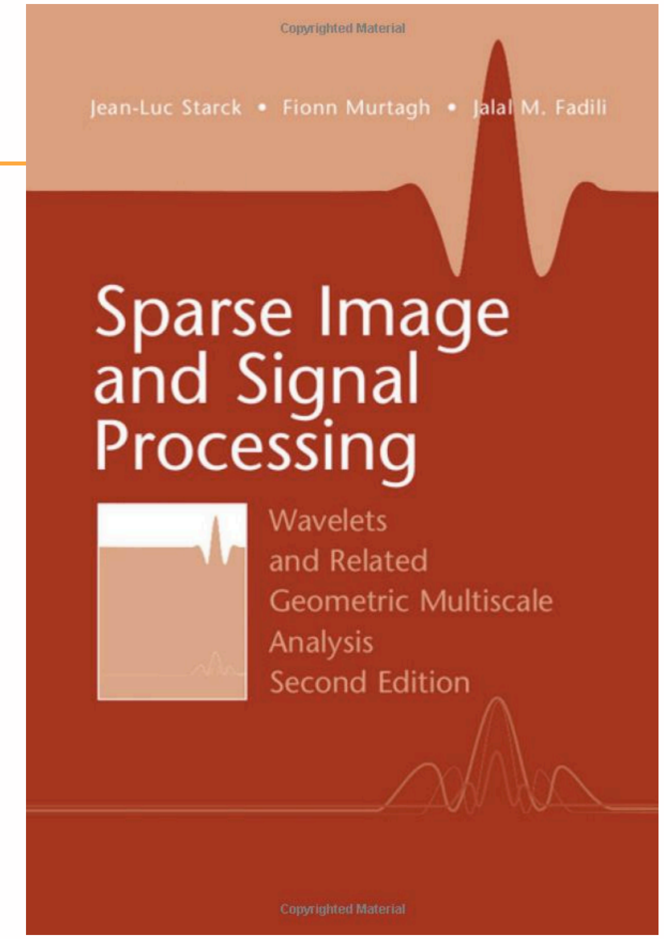
- Fast calculation of the coefficients
- Analyze the signal through the statistical properties of the coefficients
- Approximation theory uses the sparsity of the coefficients

Inverse Problems: Sparse Recovery

$$Y = HX + N$$

$$\min_X ||Y - HX||^2 \; + \; \mathcal{C}(\mathcal{X})$$

Inverse Problems: Sparse Recovery



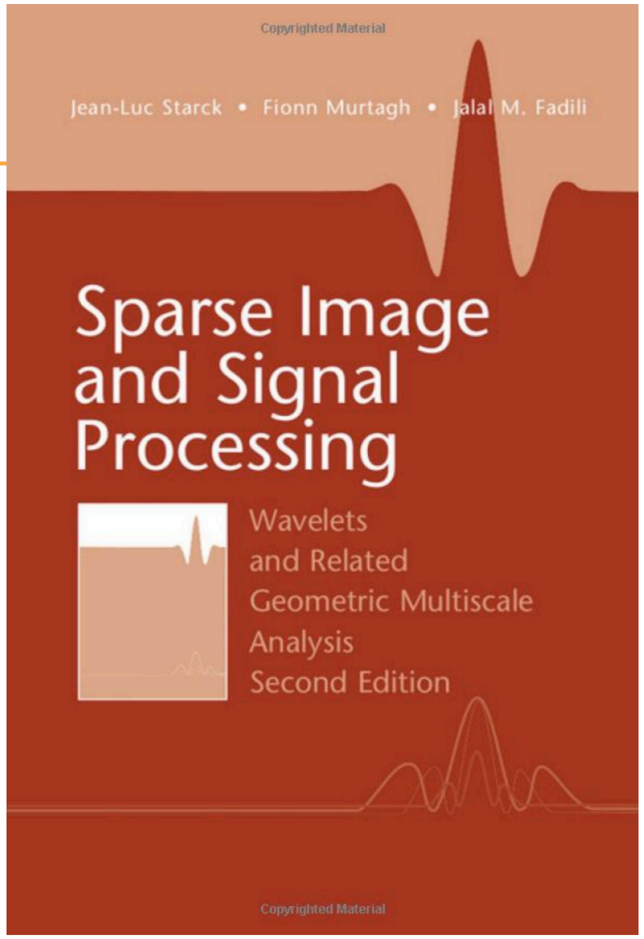
$$Y = HX + N$$

$$\min_X ||Y - HX||^2 \; + \mathcal{C}(\mathcal{X})$$

Sparse model: $X = \Phi\alpha$

$$\boxed{\min_\alpha ||Y - H\Phi\alpha||^2 + \lambda||\alpha||_p^p}$$

Inverse Problems: Sparse Recovery



$$Y = HX + N$$

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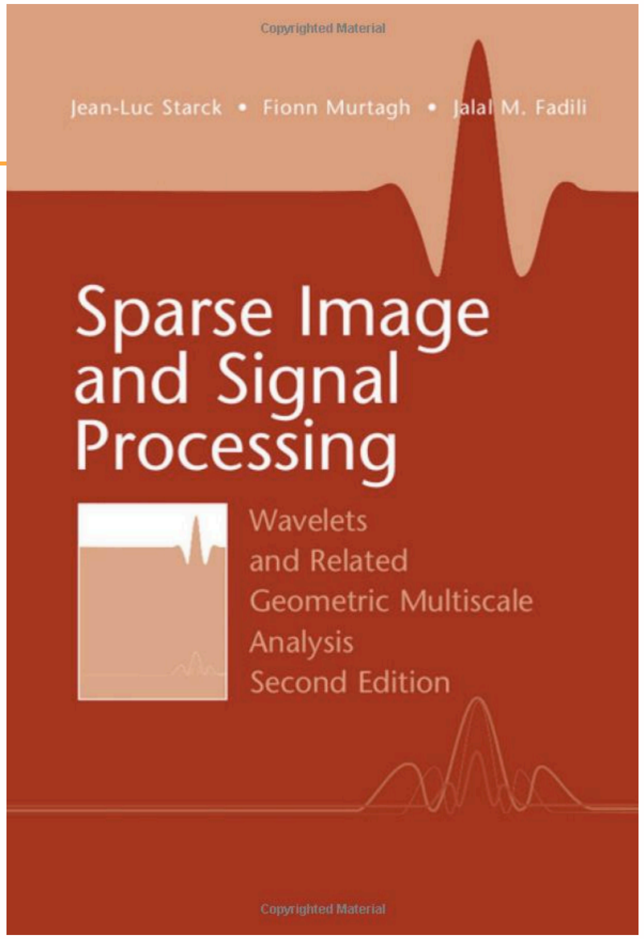
Optimization

(proximal theory)

Inverse Problems: Sparse Recovery



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Optimization

(proximal theory)

Data Representation

(harmonic analysis, machine lerning)

Inverse Problems: Sparse Recovery

$$Y = HX + N$$

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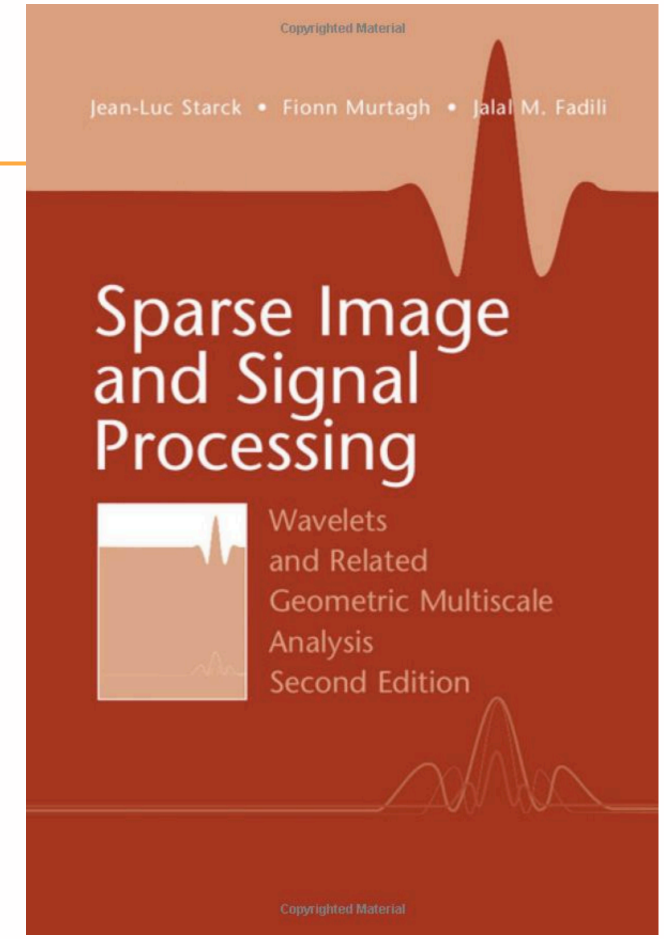


$$\boxed{\min_{\alpha} \|Y - H\Phi\alpha\|^2 + \lambda\|\alpha\|_p^p}$$

Optimization

(proximal theory)

Data Representation

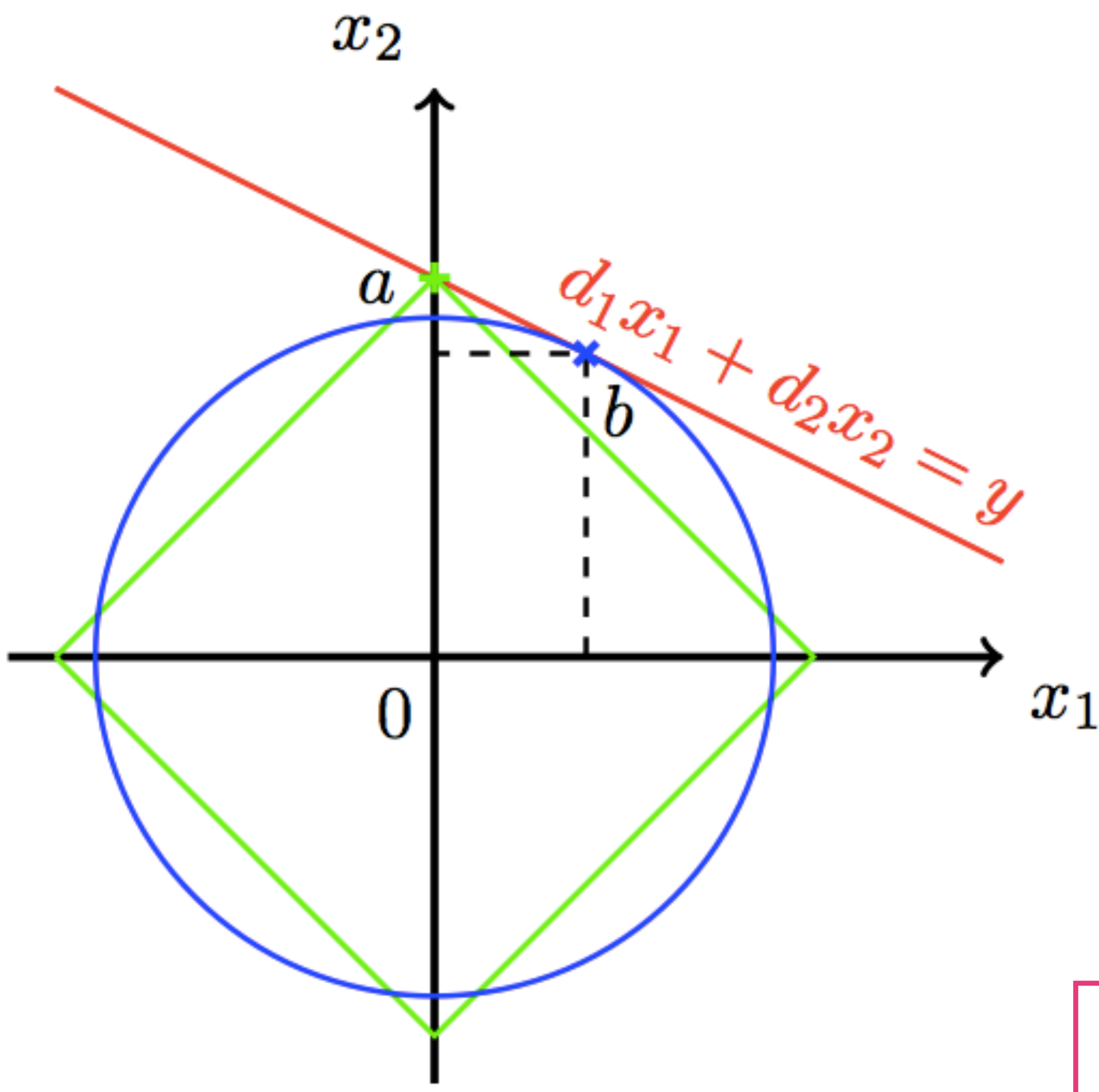
(harmonic analysis, machine lerning)

Noise Modeling

(Gaussian, Poissson, etc)

L1 Norm & Sparsity

$$\|X\|_p = \left(\sum_i | X_i |^p\right)^{\frac{1}{p}}$$



$$\boxed{p < 2}$$

Compressed Sensing

* E. Candès and T. Tao, "Near Optimal Signal Recovery From Random Projections: Universal Encoding Strategies? ", IEEE Trans. on Information Theory, 52, pp 5406-5425, 2006.
* D. Donoho, "Compressed Sensing", IEEE Trans. on Information Theory, 52(4), pp. 1289-1306, April 2006.
* E. Candès, J. Romberg and T. Tao, "Robust Uncertainty Principles: Exact Signal Reconstruction from Highly Incomplete Frequency Information", IEEE Trans. on Information Theory, 52(2) pp. 489 - 509, Feb. 2006.

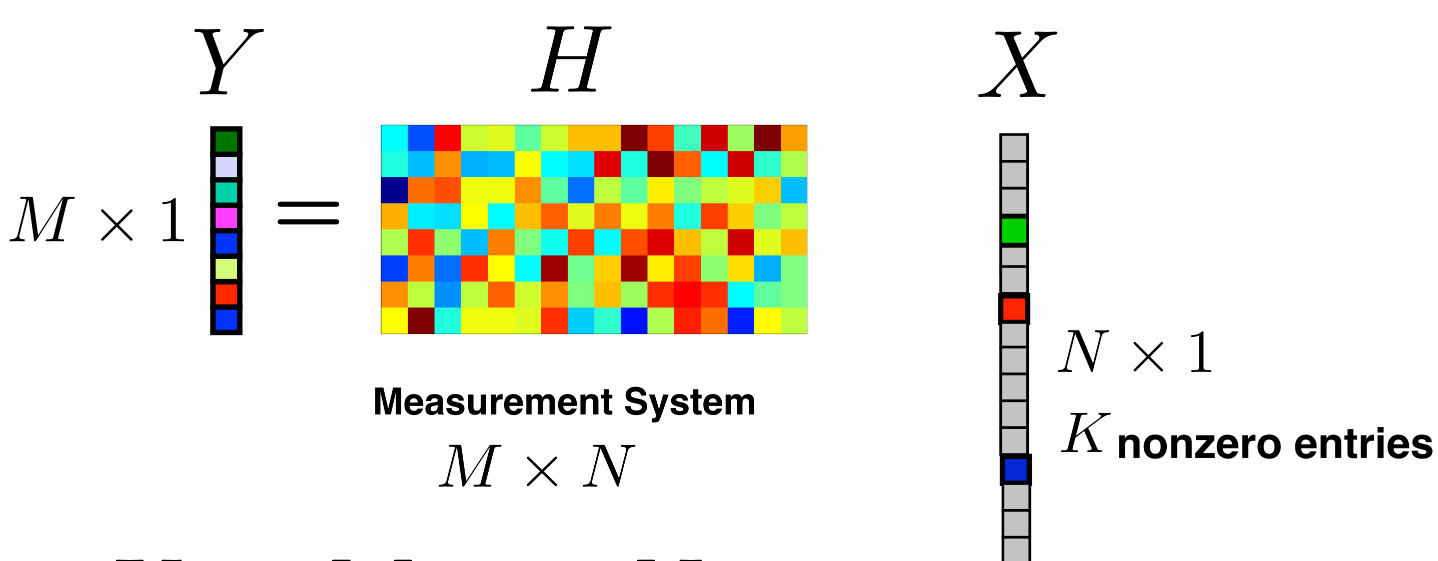
A non linear sampling theorem

"Signals with exactly K components different from zero can be recovered perfectly from ~ K log N incoherent measurements"

Replace samples with *few linear projections* $Y = HX$

Incoherent Measurements

X = K sparse signal



$$K < M << N$$

Reconstruction via non linear processing: $\min_X \| X \|_1 \quad \text{s.t.} \quad Y = HX$

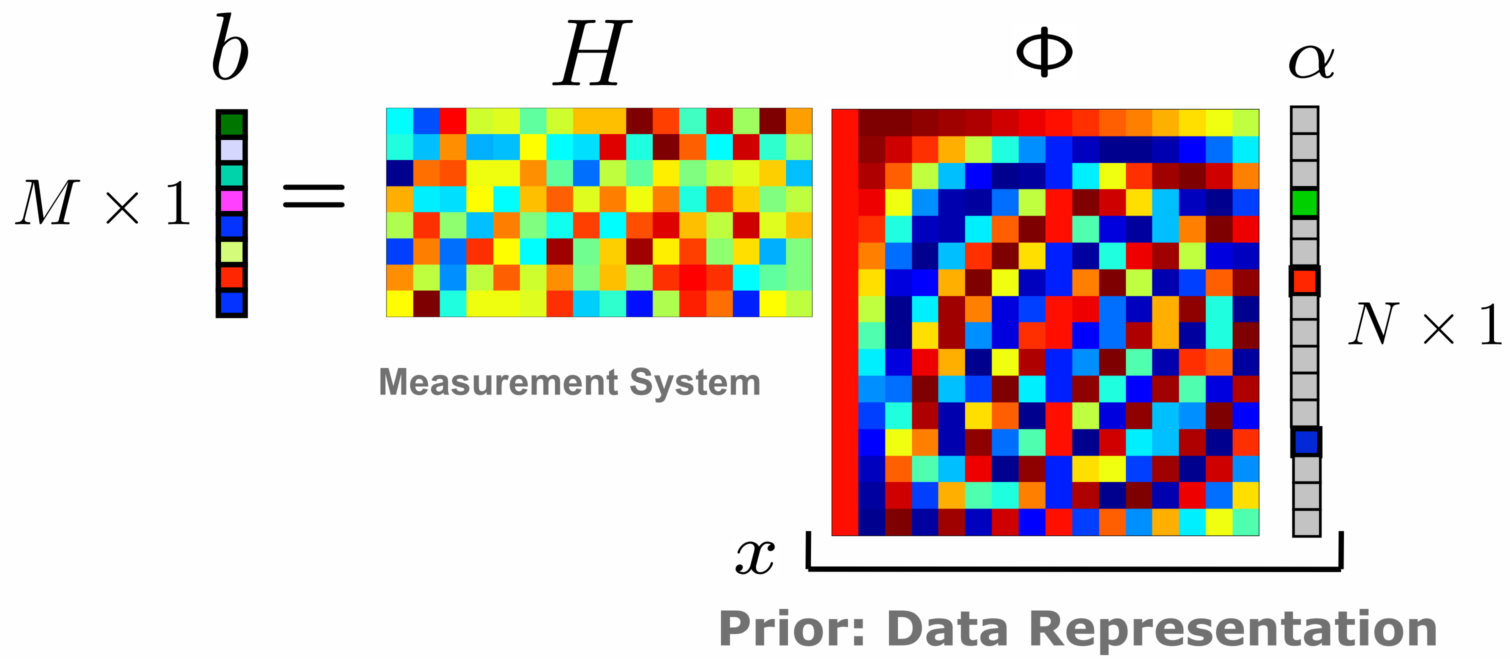
Mutual Incoherence

In practice, X is sparse in a **dictionary**: $X = \Phi\alpha$

optimally incoherent

ex: Fourier/Dirac

perfectly coherent ensembles



Incoherence between a sparse representation and a measurement matrix (i.e. mutual incoherence):

$$\mu(H, \mathbf{\Phi}) = \max_{i,j} |\langle H_i, \phi_j \rangle|$$

measures how an atom of the sparse representation spreads in the measurement ensemble.

$$\frac{1}{\sqrt{n}} \leq \mu(H, \mathbf{\Phi}) \leq 1$$

Mutual Incoherence

In the case of **exactly K-sparse signals**, and **perfect measurements** (no noise), the following result holds:

If $$m \geq C\, \mu(\mathbf{H}, \mathbf{\Phi})^2\, K \log(n)$$

Then $$\min_{\alpha} ||\alpha||_{\ell_1} \ \text{s.t.}\ \ y = \mathbf{H}\mathbf{\Phi}\alpha$$

Provides a perfect reconstruction

Effective Sparse Recovery algorithms

CS recovery requires solves convex but non-smooth minimization problems

$$\min_{\alpha} \|Y - H\Phi\alpha\|^2 + J(\alpha)$$

sparsity penalization in Φ

Since a decade, proximal algorithms can solve efficiently these problems, even in large-scale settings:

$$X^{(n+1)} = \underbrace{\mathrm{prox}_{\gamma J}}_{\textit{Proximal operator}} \left(\underbrace{X^{(n)} + \gamma \boldsymbol{H}^T (Y - \boldsymbol{H} X^{(n)})}_{\textit{Gradient descent step}} \right)$$

Ex. Forward splitting Algorithm (Combettes et al. 2005), FISTA (Beck & Teboulle 2009), etc.

Sparse Recovery & Inverse Problems

$$Y = HX + N \qquad X = \Phi\alpha$$

and α is **sparse** or **compressible**

$$\boxed{\min_{\alpha} ||Y - H\Phi\alpha||^2 + \lambda||\alpha||_p^p}$$

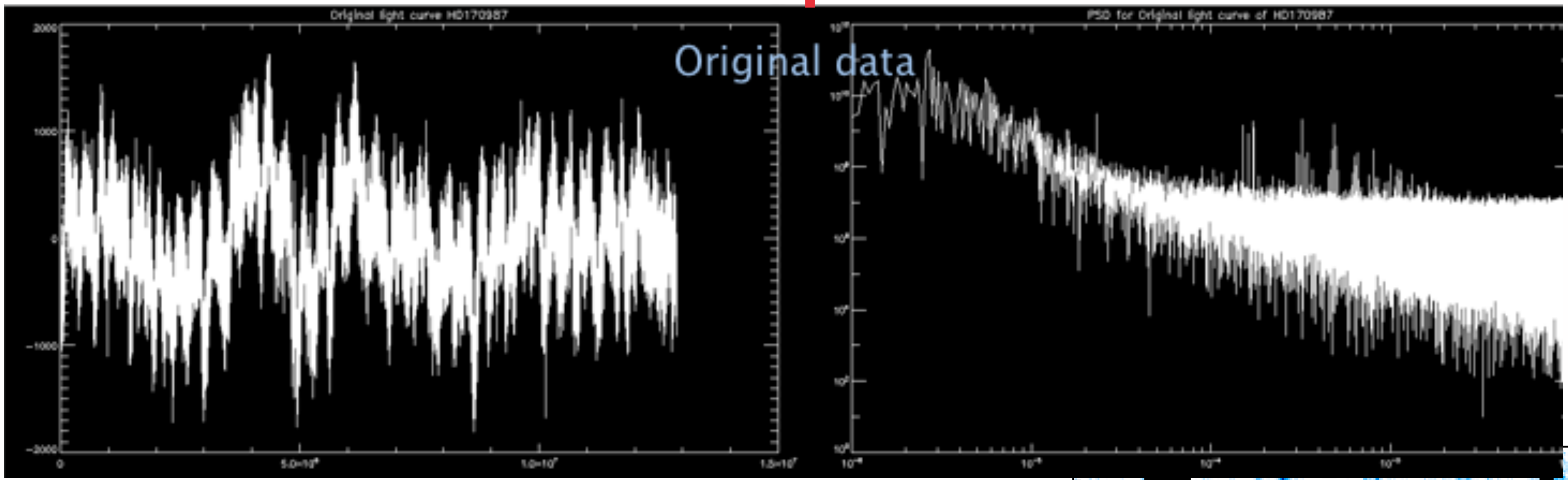
- Denoising
- Deconvolution
- Component Separation
- **Inpainting**
- Blind Source Separation
- Minimization algorithms
- Compressed Sensing

Interpolation of Missing Data

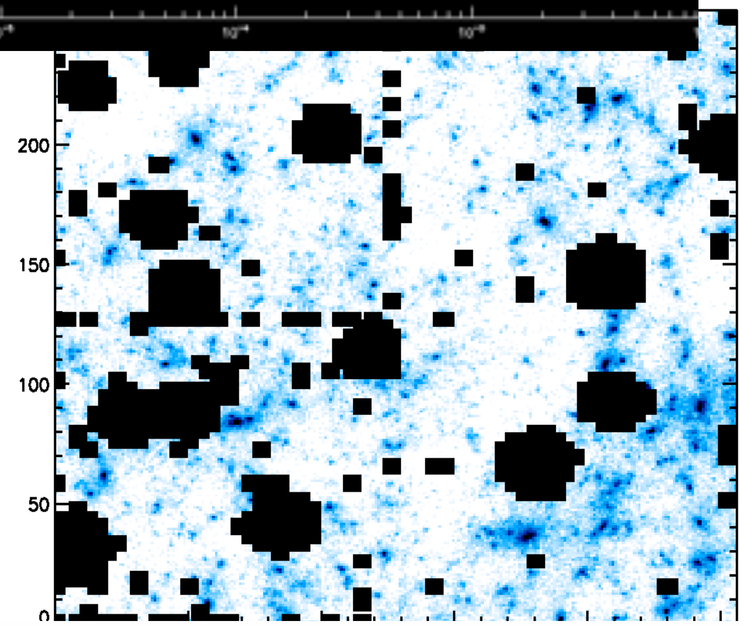
Inpainting

M. Elad, J.-L. Starck, D.L. Donoho, P. Querre, “Simultaneous Cartoon and Texture Image Inpainting using Morphological Component Analysis (MCA), ACHA, Vol. 19, pp. 340-358, 2005.

- Period detection in temporal series



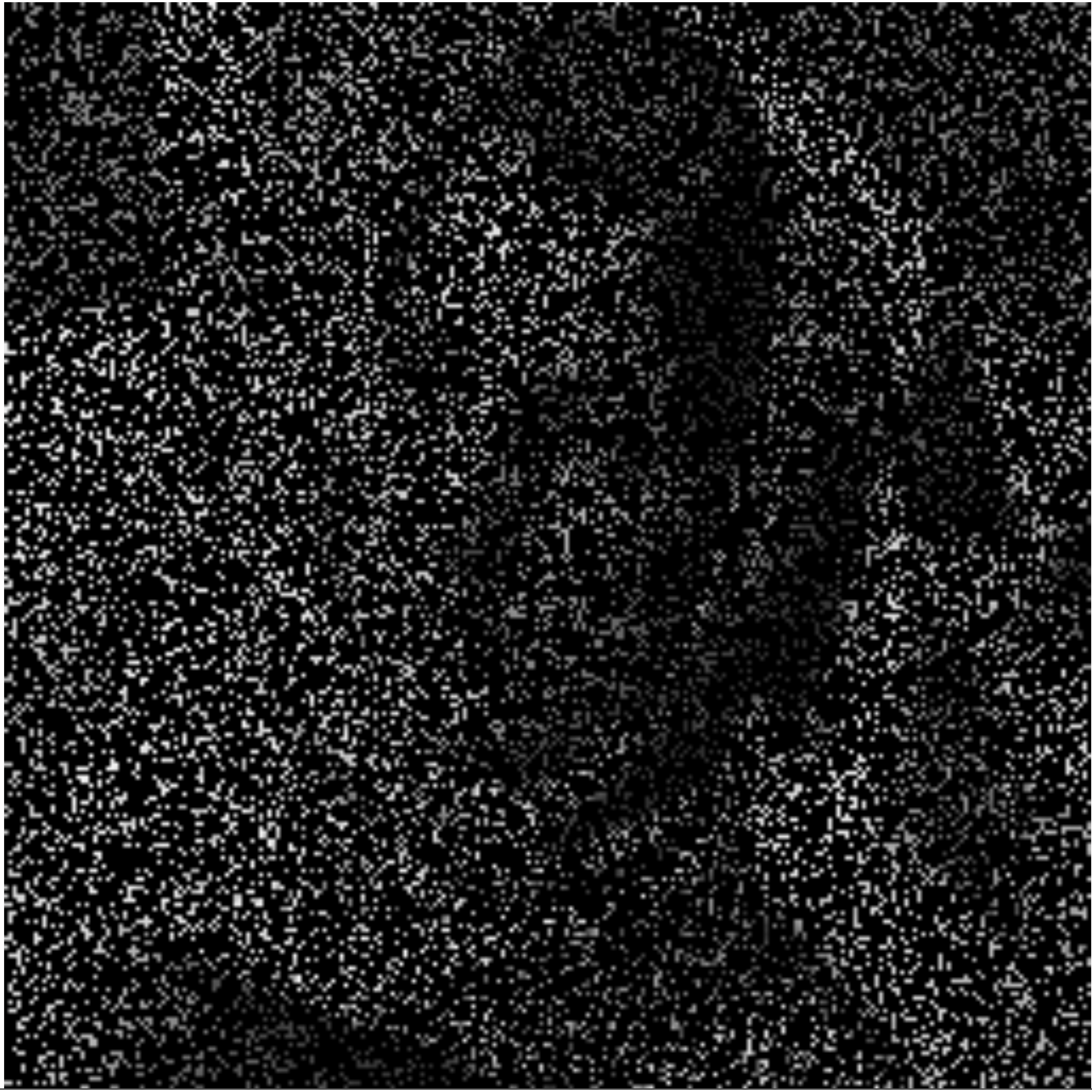
- Bad pixels, cosmic rays, point sources in 2D images, ...



20%

20%

80%



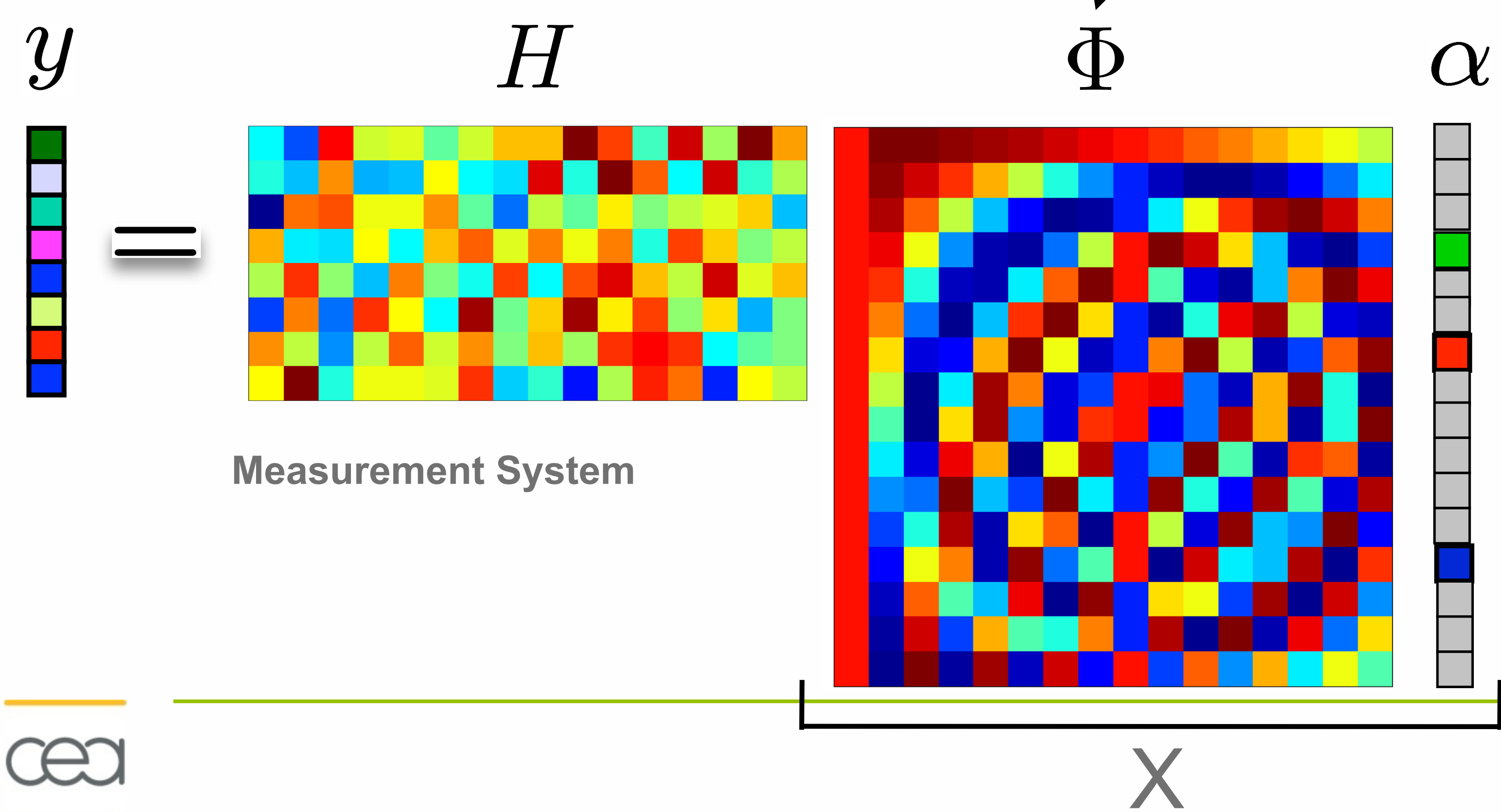
80%

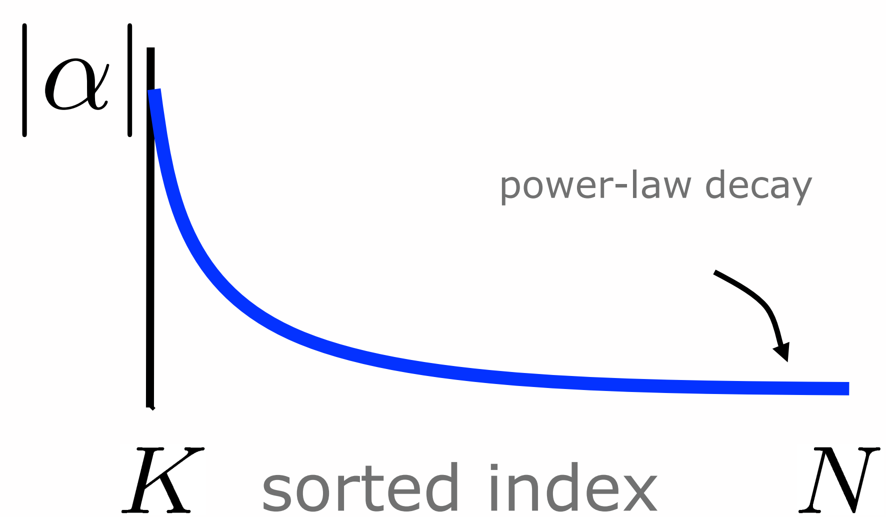
The Dictionary

$Y = HX + N$ $\quad X = \Phi\alpha$ and α is **sparse** or **compressible**

$$\mu(H, \mathbf{\Phi}) = \max_{i,j} |\langle H_i, \phi_j \rangle|$$

$$\boxed{\min_{\alpha} \|Y - H\Phi\alpha\|^2 + \lambda \|\alpha\|_p^p}$$



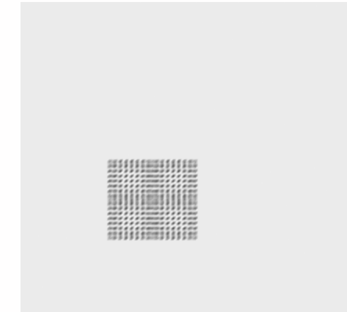


Sparse Representations

Local DCT

Stationary textures

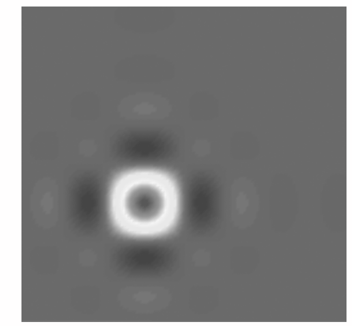
Locally oscillatory



Wavelet transform

Piecewise smooth

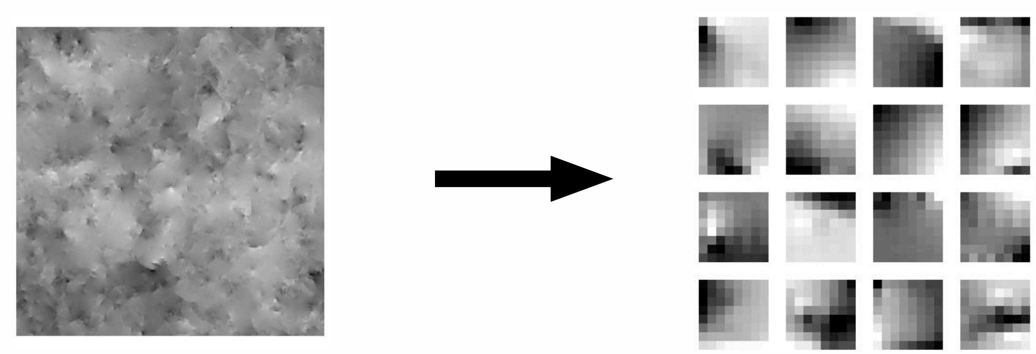
Isotropic structures



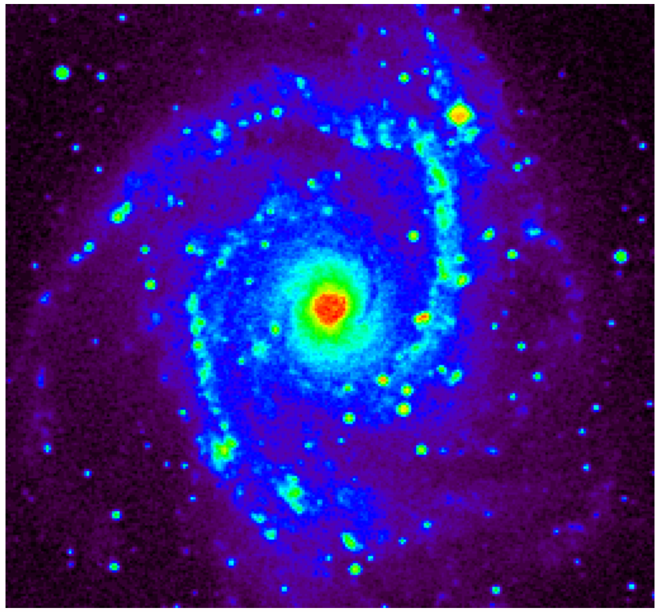
Curvelet transform

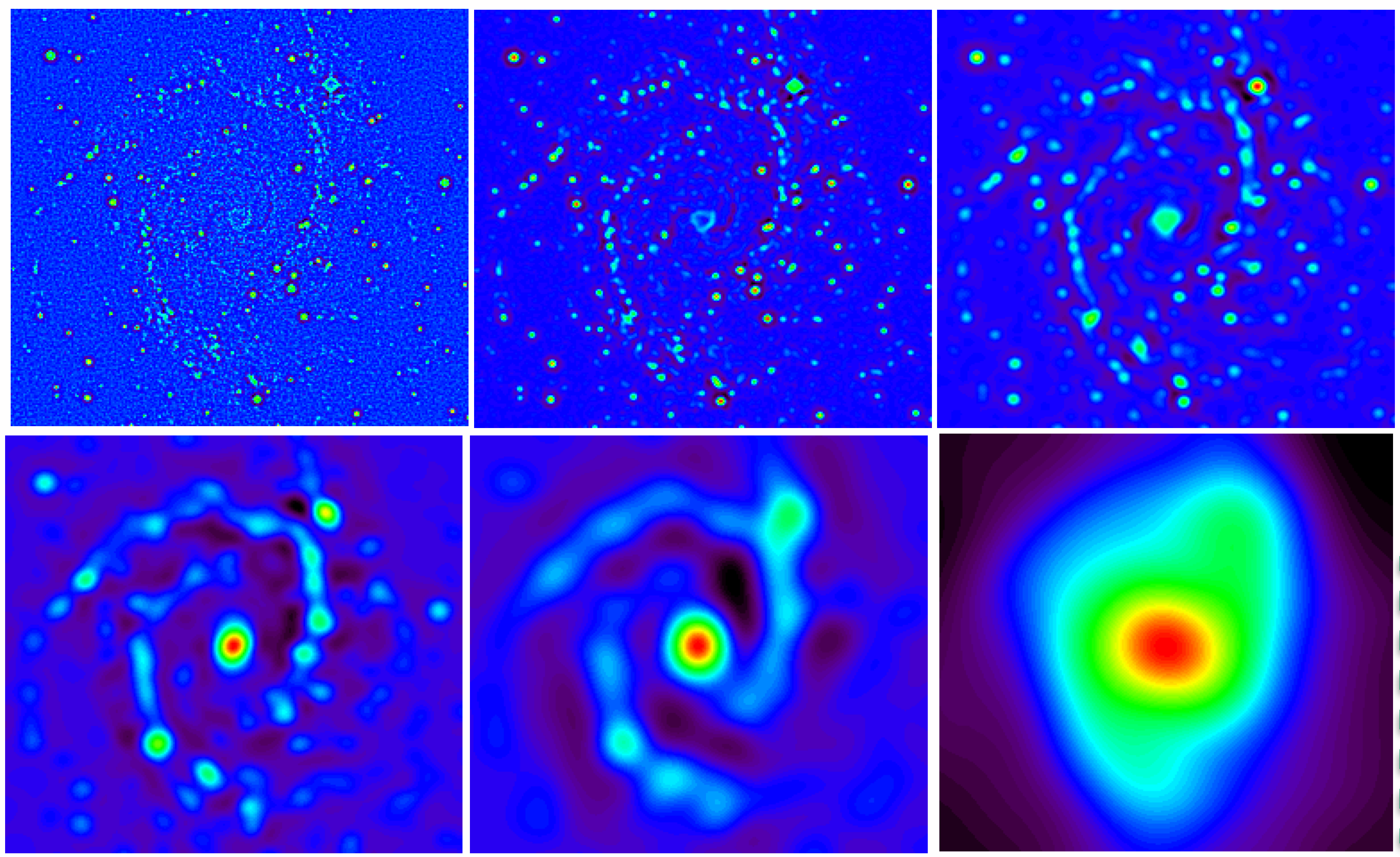
Piecewise smooth,
edge

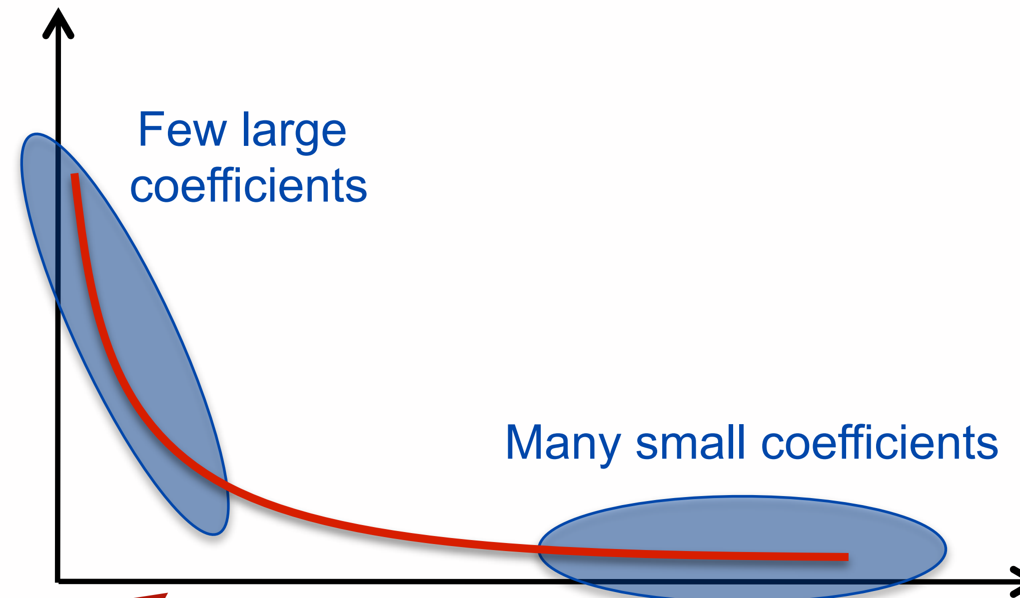
Dictionary Learning



The STARLET Transform







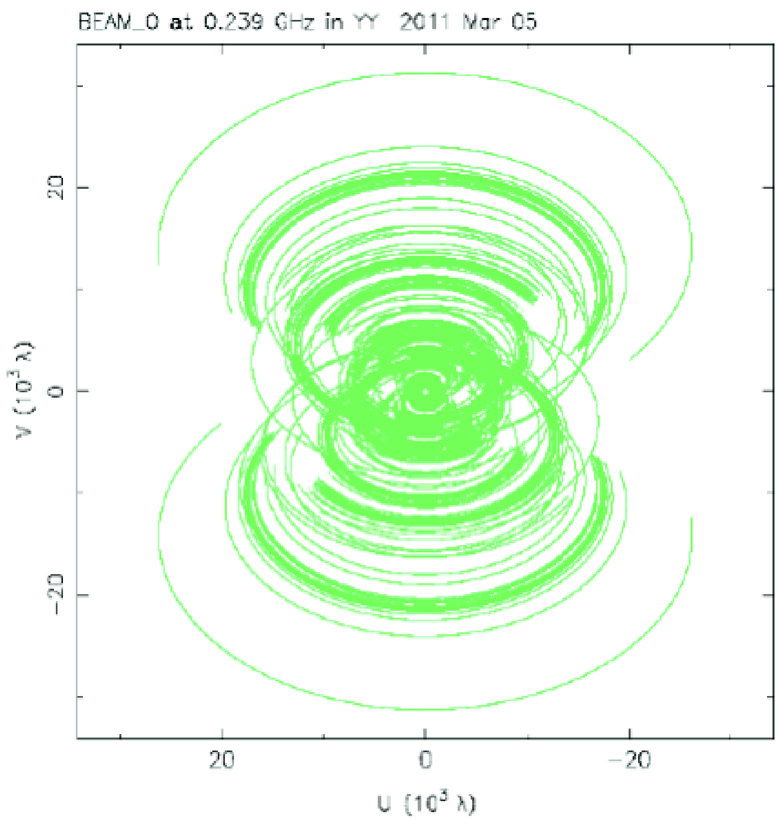
Sorted wavelet coefficients

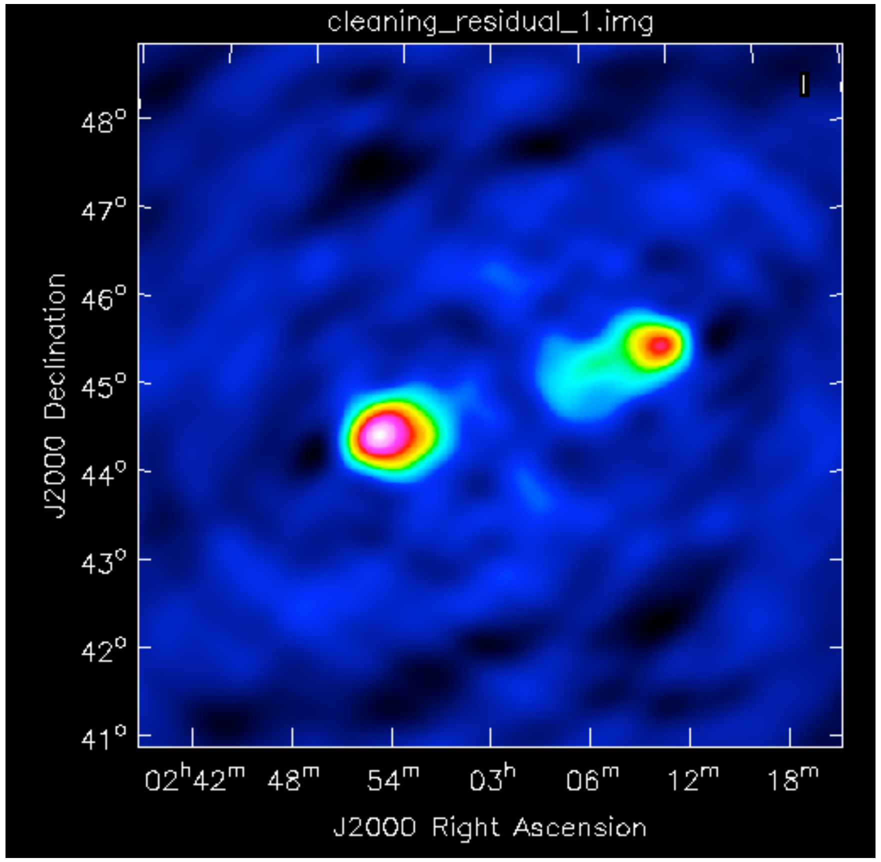
Sparse Model :

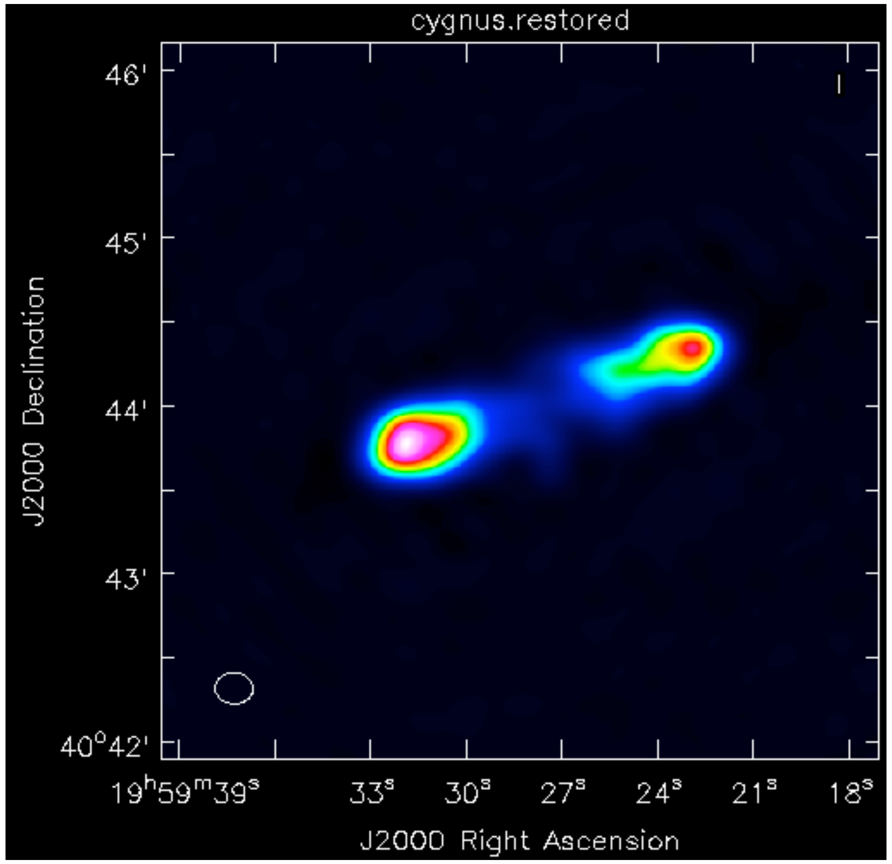
x is approximately sparse in Φ

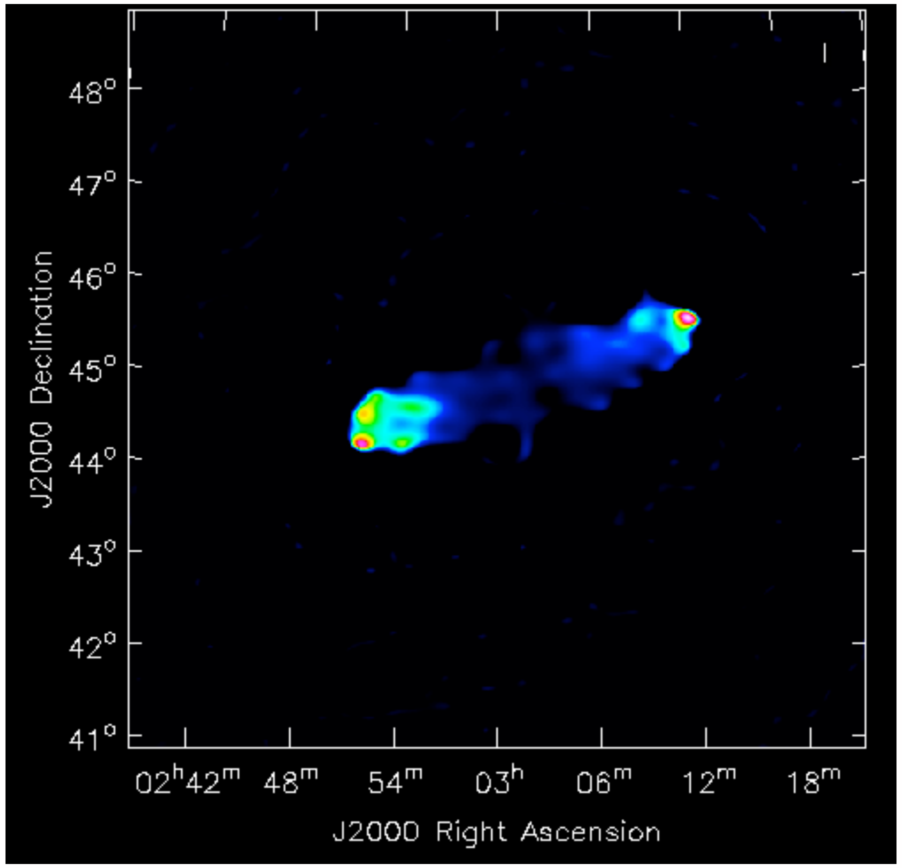
Starlet transform
(isotropic undecimated wavelet transform)

Compressed Sensing & LOFAR Cygnus A Data









Garsden et al, "**LOFAR Image Sparse Reconstruction**", A&A, 575, A90, 2015.

http://arxiv.org/abs/1406.7242

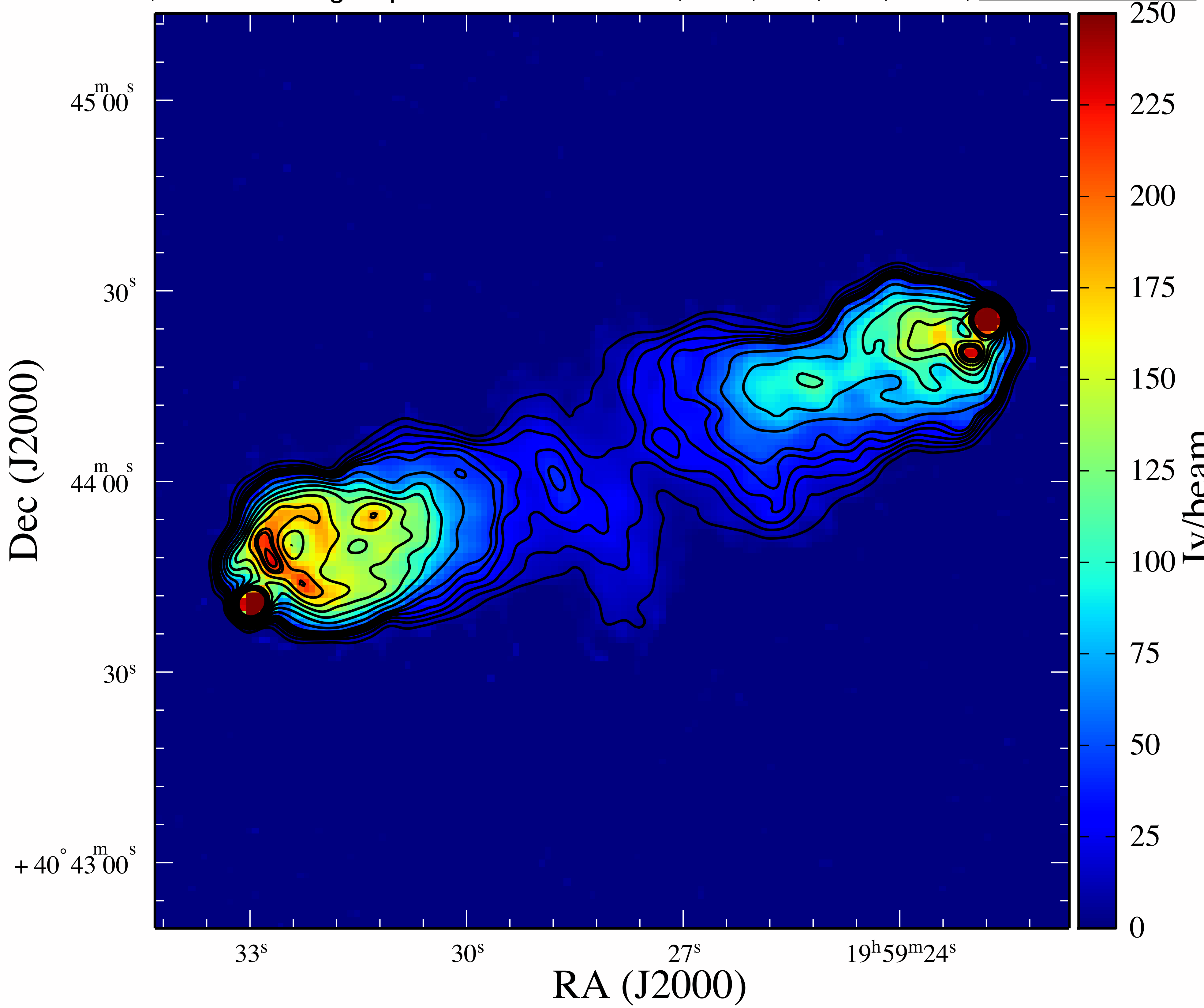
Garsden et al, "LOFAR Image Sparse Reconstruction", A&A, 575, A90, 2015, ArXiv:1406.7242.

J. Girard

H. Garsden

S. Corbel

C. Tasse



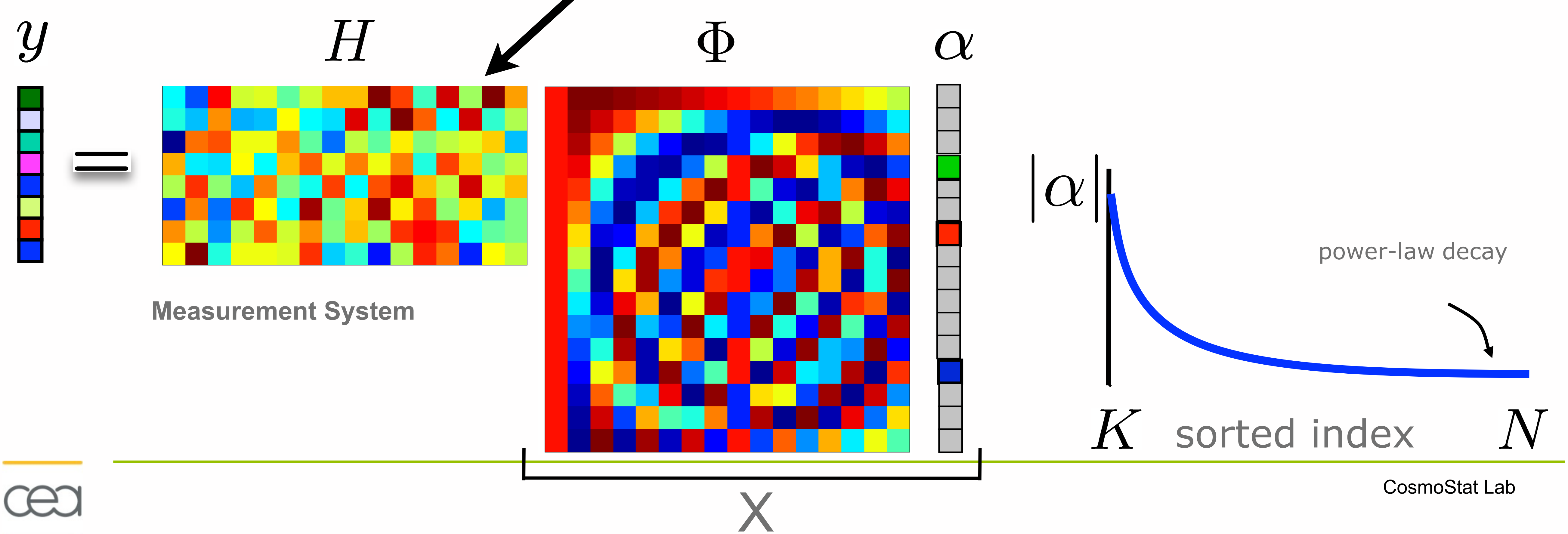
Colorscale: reconstructed 512x512 image of Cygnus A at 151 MHz (with resolution 2.8" and a pixel size of 1"). Contours levels are [1,2,3,4,5,6,9,13,17,21,25,30,35,37,40] Jy/Beam from a 327.5 MHz Cyg A VLA image (Project AK570) at 2.5" angular resolution and a pixel size of 0.5". **Recovered features in the CS image correspond to real structures observed at higher frequencies.**

The Acquisition Matrix

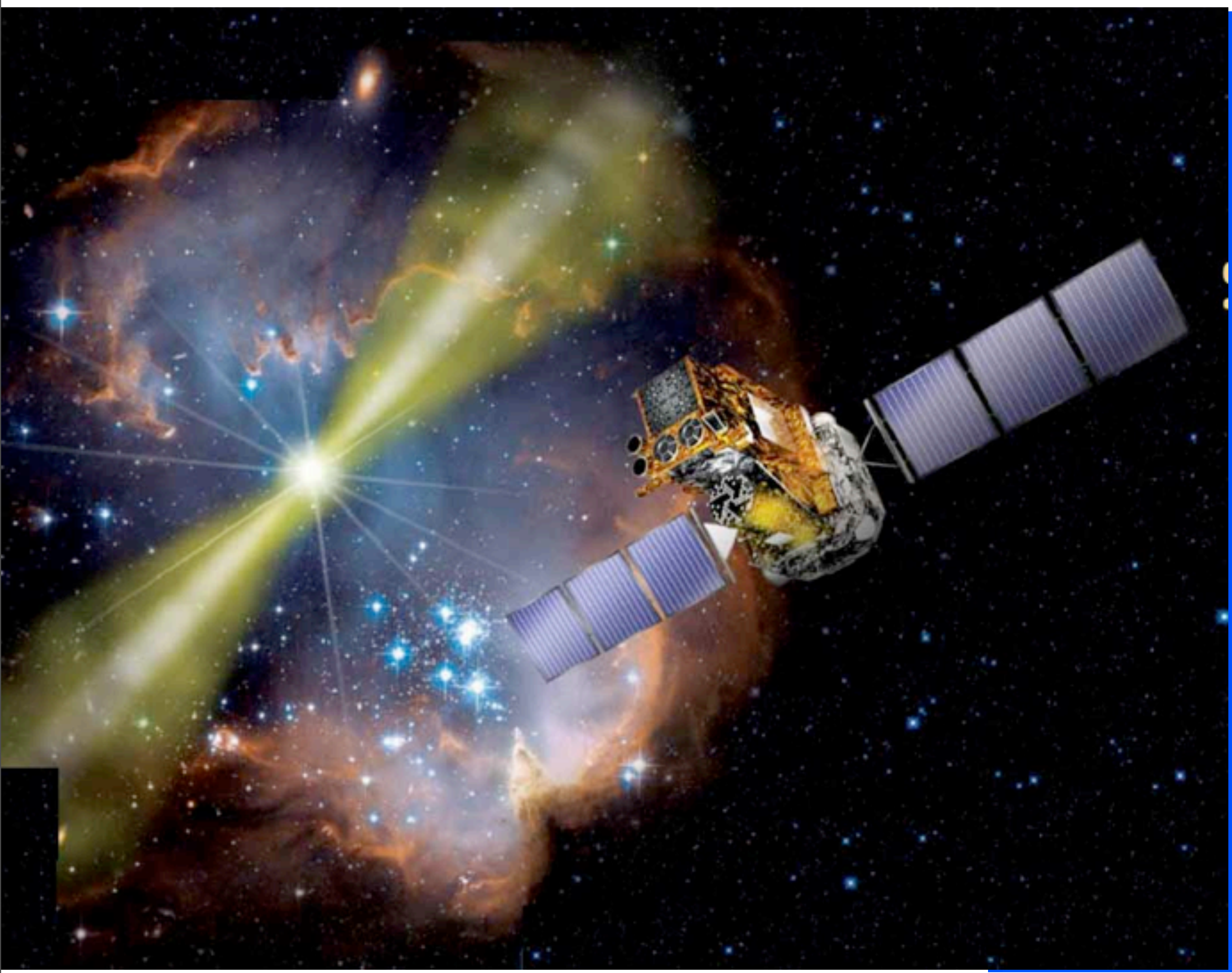
$Y = HX + N \qquad X = \Phi\alpha$ and α is **sparse** or **compressible**

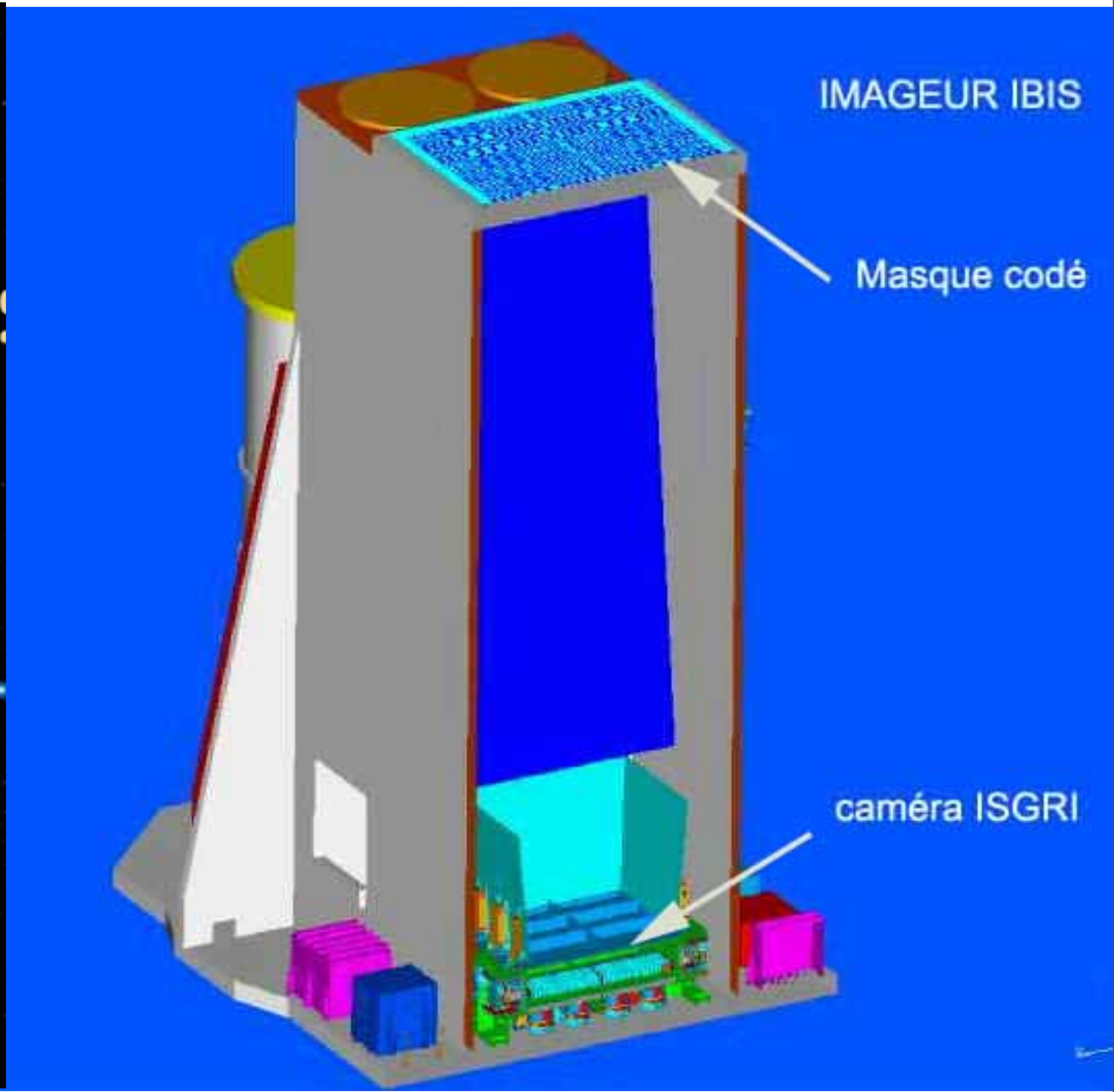
$$\mu(H, \mathbf{\Phi}) = \max_{i,j} |\langle H_i, \phi_j \rangle|$$

$$\boxed{\min_{\alpha} \|Y - H\Phi\alpha\|^2 + \lambda\|\alpha\|_p^p}$$



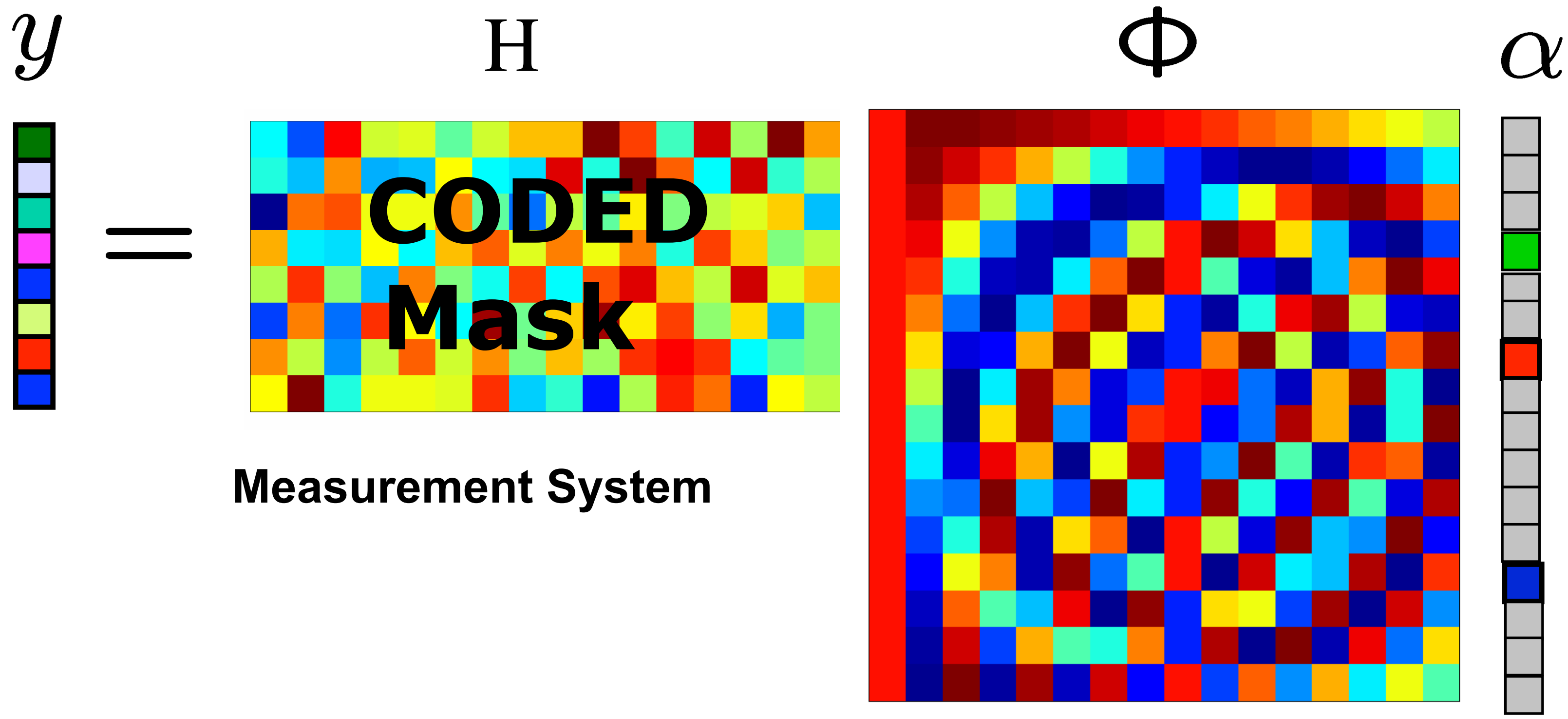
IBIS Camera on board of the Integral satellite

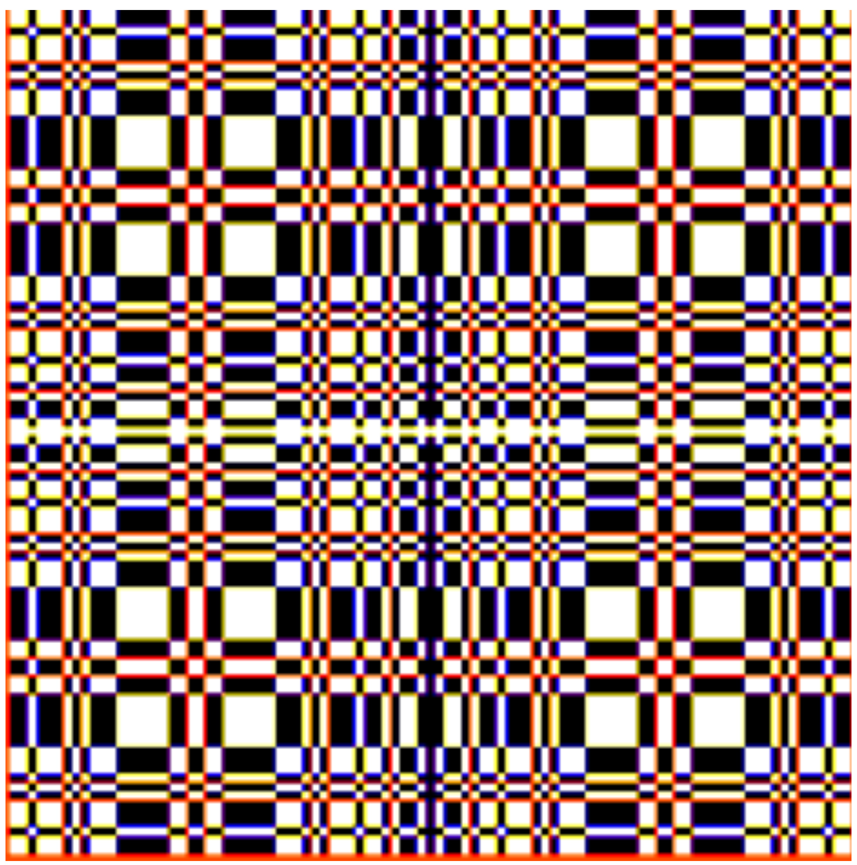




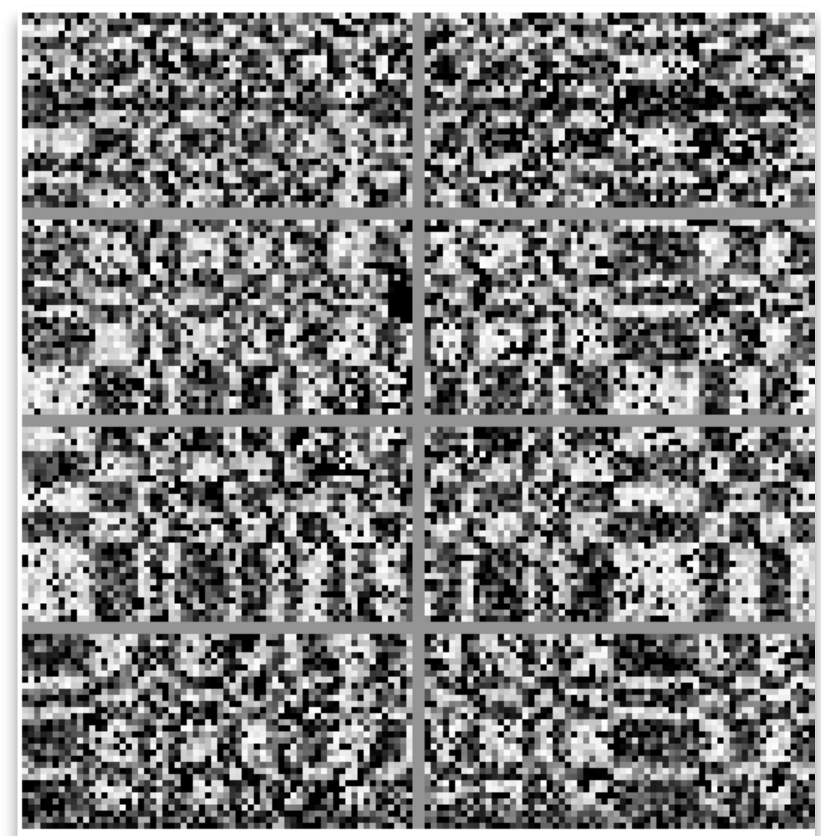
Launch, October 4, 2002.

Gamma Ray Instruments (Integral) - Acquisition with coded masks





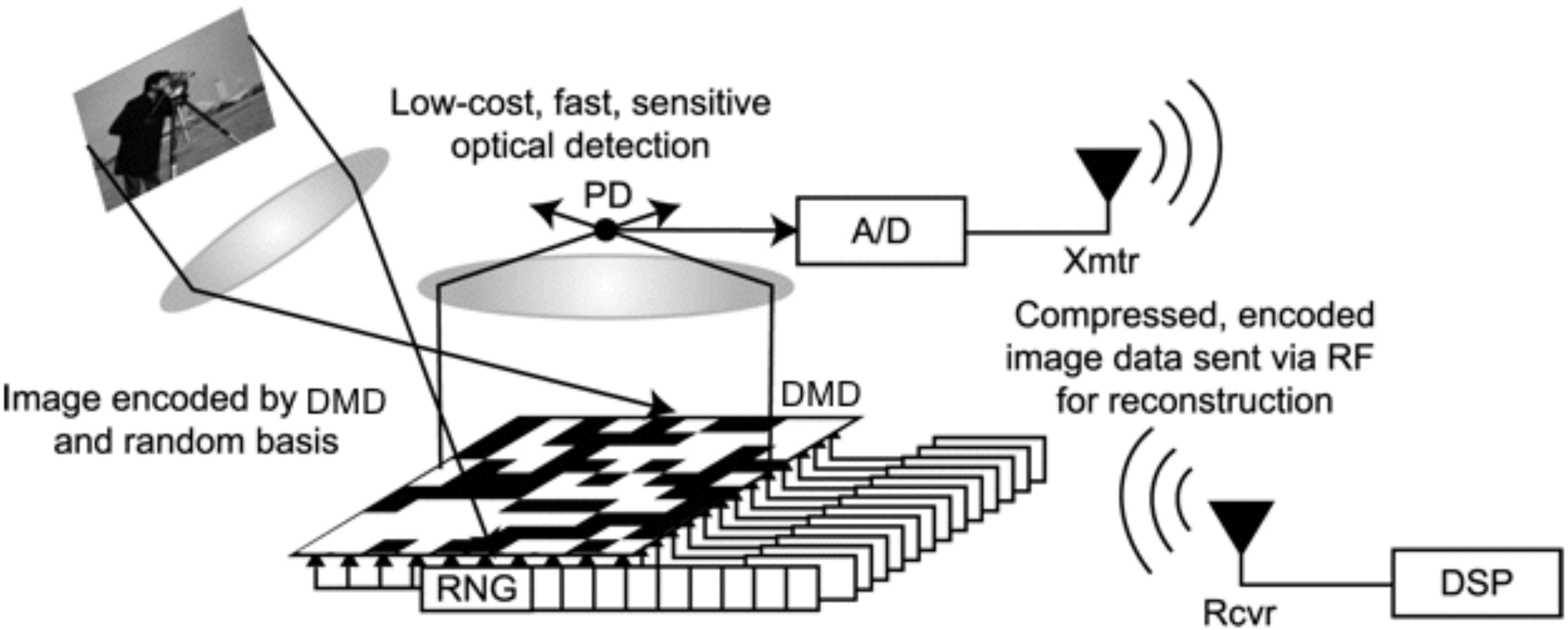
INTEGRAL/IBIS Coded Mask



Crab Nebula Integral Observation

Courtesy I. Caballero, J. Rodriguez (AIM/Saclay)

Single Pixel Camera (Rice Universty)
http://dsp.rice.edu/cscamera



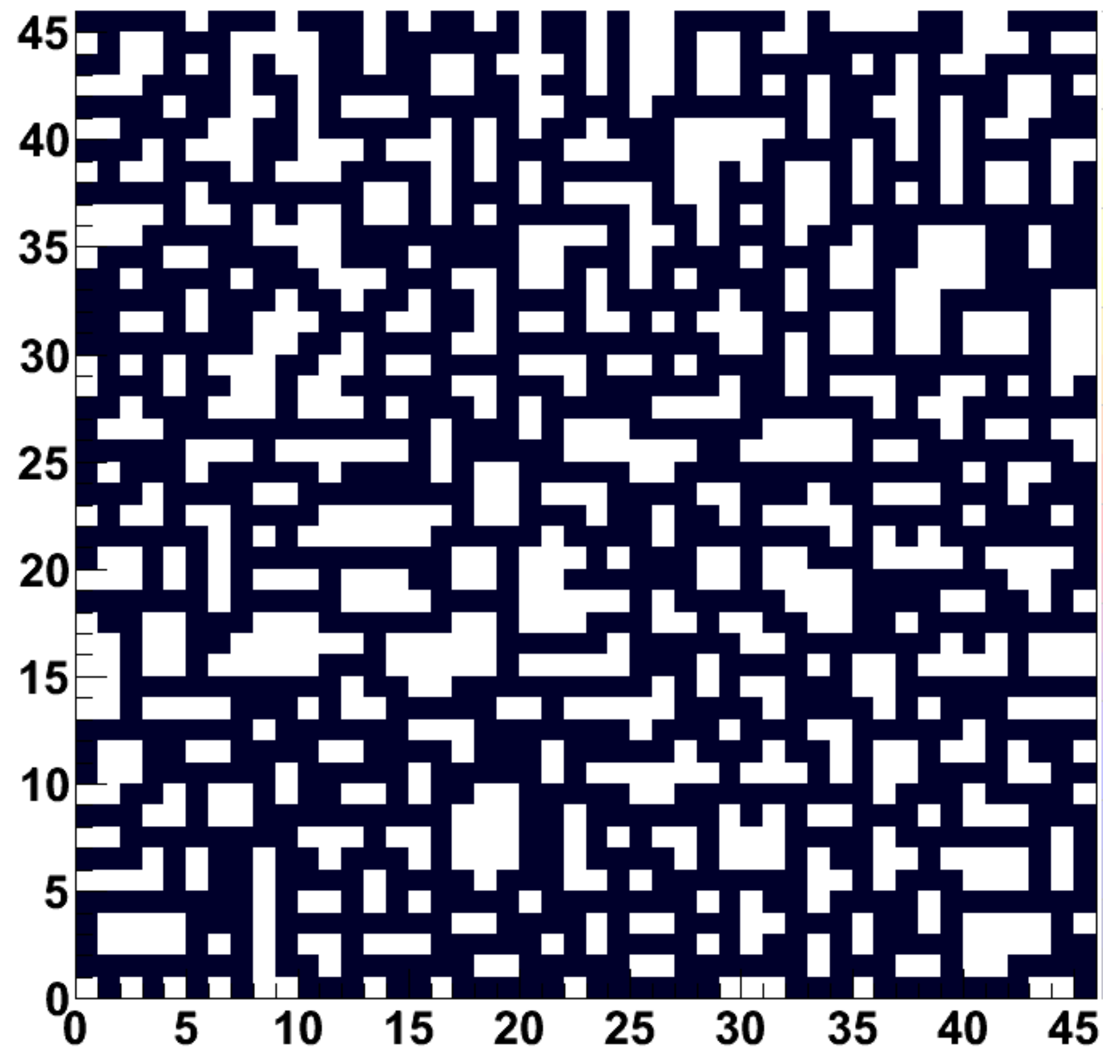
x

SVOM (future French-Chinese Gamma-Ray Burst mission)

saclay
irfu

- ***ECLAIRs france-chinese satellite 'SVOM'***
  Gamma-ray detection in energy range 4 - 120 keV
  Coded mask imaging (at 460 mm of the detector plane)

Physical mask pattern
(46 x 46 pixels of 11.7 mm)

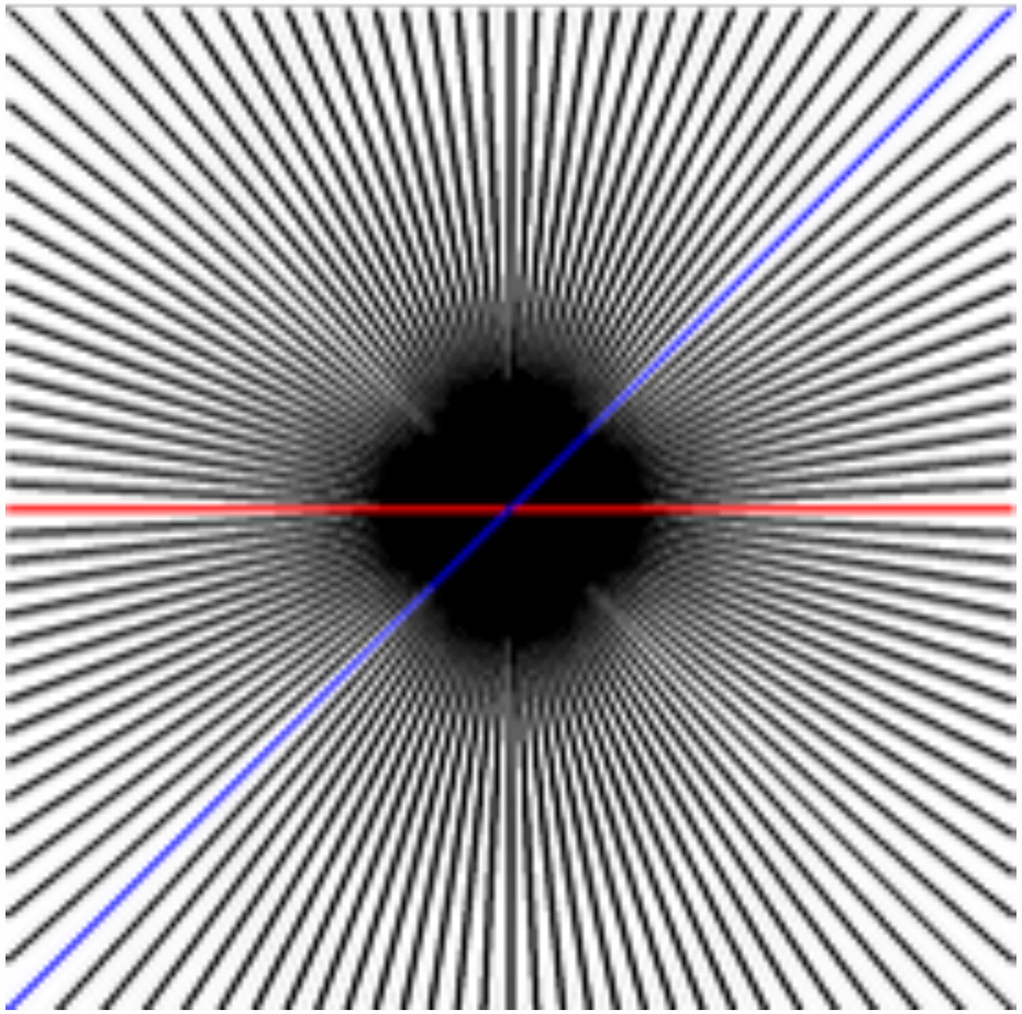


Stéphane Schanne – CEA

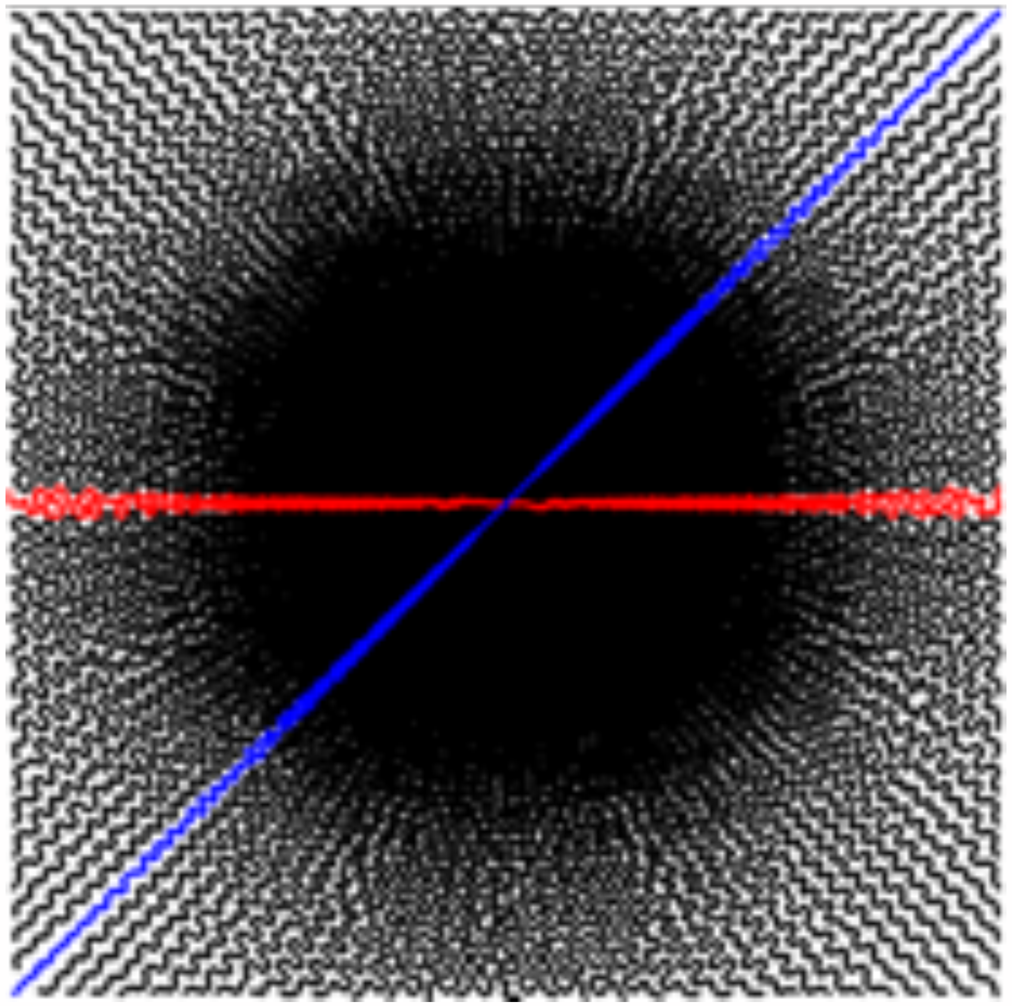
Design of new sampling patterns (k-space trajectories)

C. Lazarus et al, "*SPARKLING: Novel non-Cartesian sampling schemes for accelerated 2D anatomical imaging at 7T using compressed sensing*", in Proc. 25th Scientific Meeting, Int. Society for Magnetic Resonance in Medicine, 2017.

RADIAL



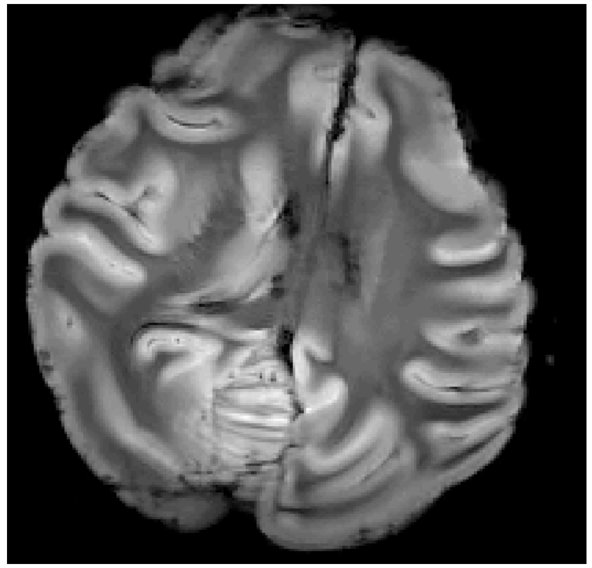
SPARKLING



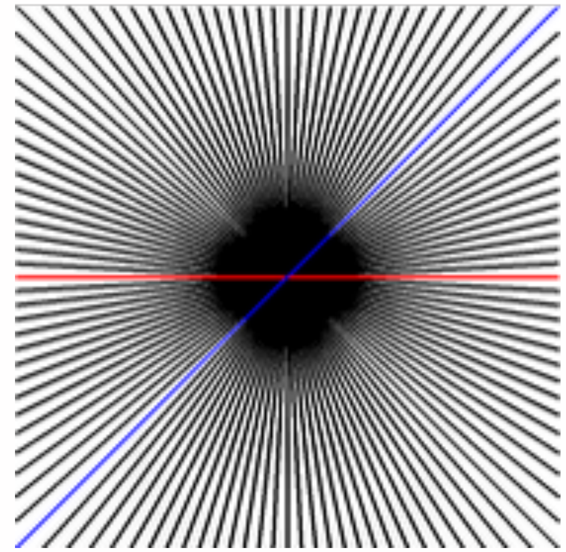
=> See Carole's Poster

Sparkling

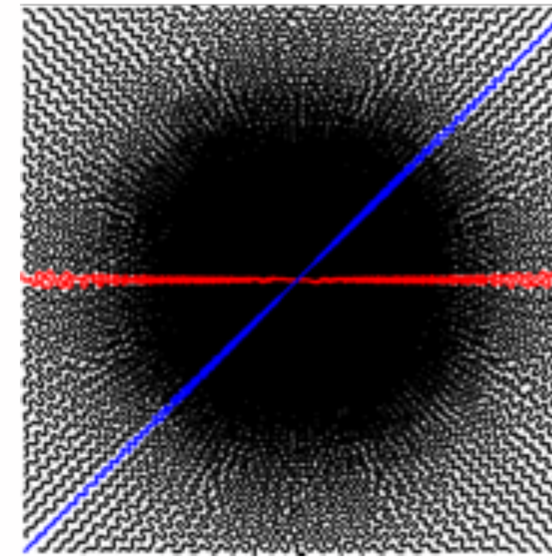
- **Sequence parameters:**
- - N=512
- - FOV=200x200 mm²
- - TR=550 ms and TE=30 ms
- - α=25°
- - BW=32.55 Hz/px
- - **Tobs=30.72 ms**
- - Slice thickness: 3 mm
- - Single channel receiver coil
- - Ex-vivo baboon brain



RADIAL



SPARKLING



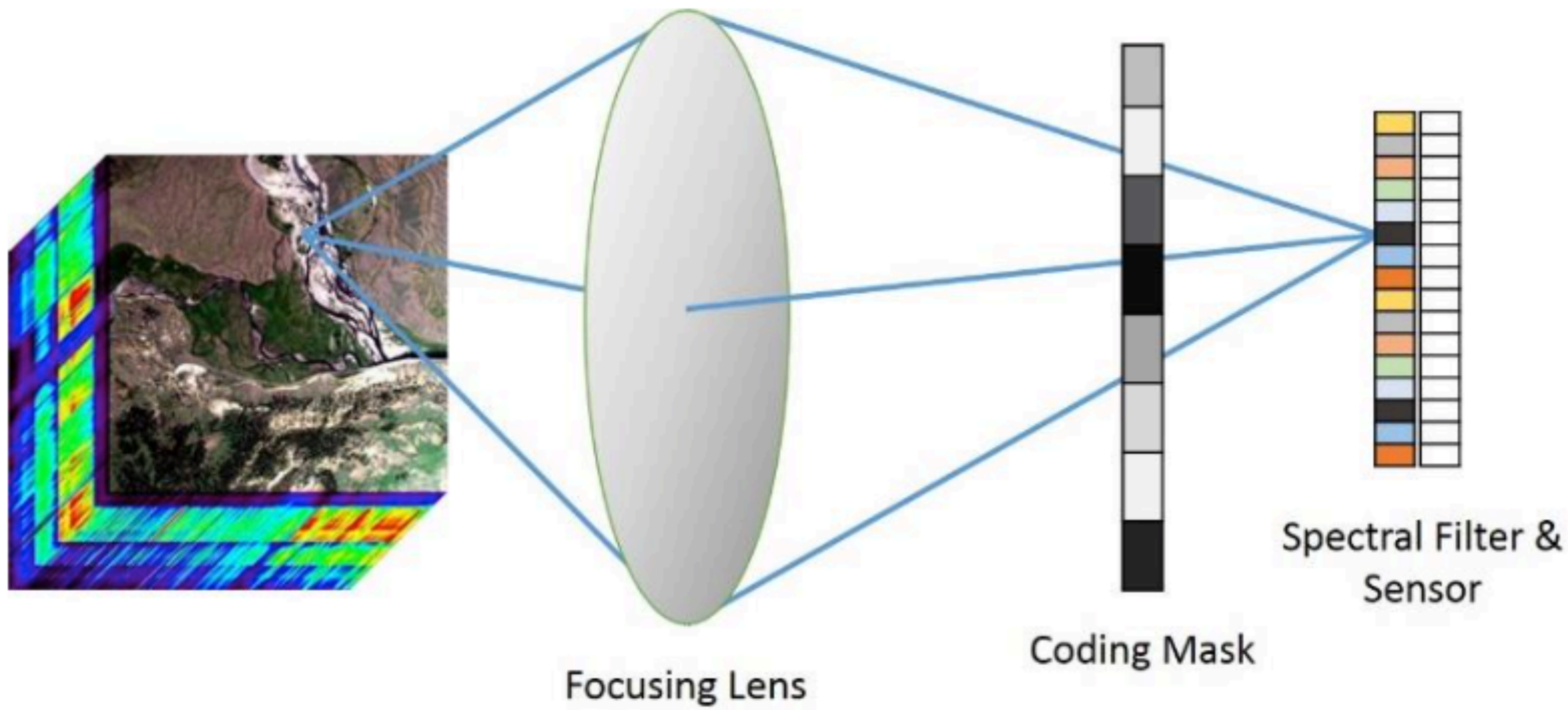
λ=2.10^{-5}

Acceleration factor = 8.5

Acceleration factor = 8.5

CosmoStat Lab

Perspective: CS Hardware for Hyperspectral Imaging

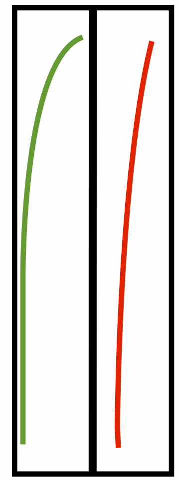


H2020 HyperSPACE submitted project for CS remote sensing

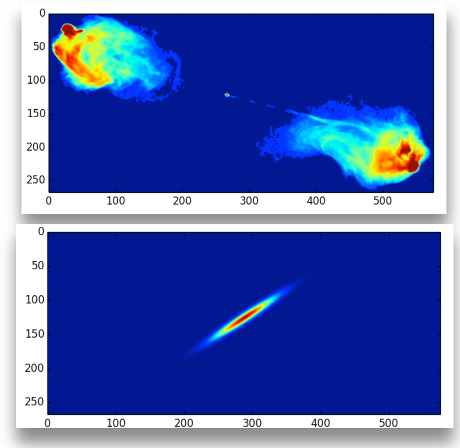
Hyperspectral Data

Ground Truth

A

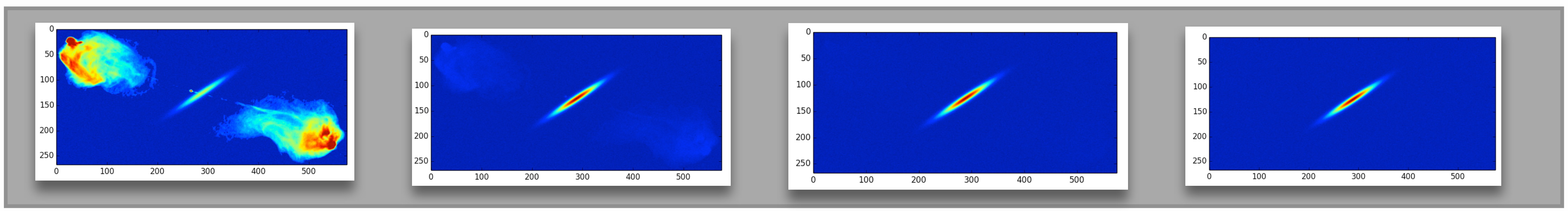


E



Mixtures

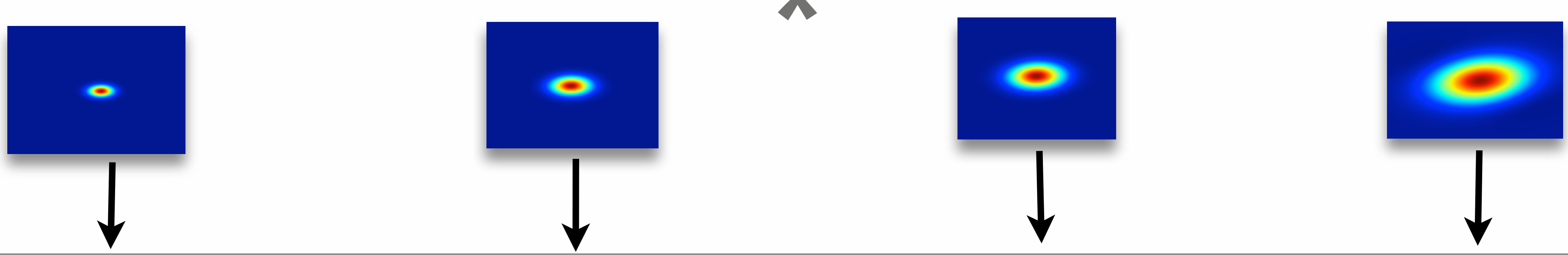
$X = AE$



chan 1 chan 4 chan 7 chan 10

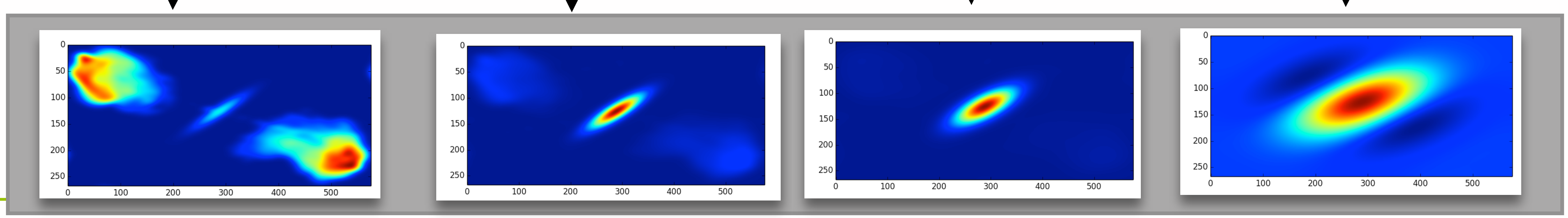
$*$

PSF H



Data

$Y = HX + N$

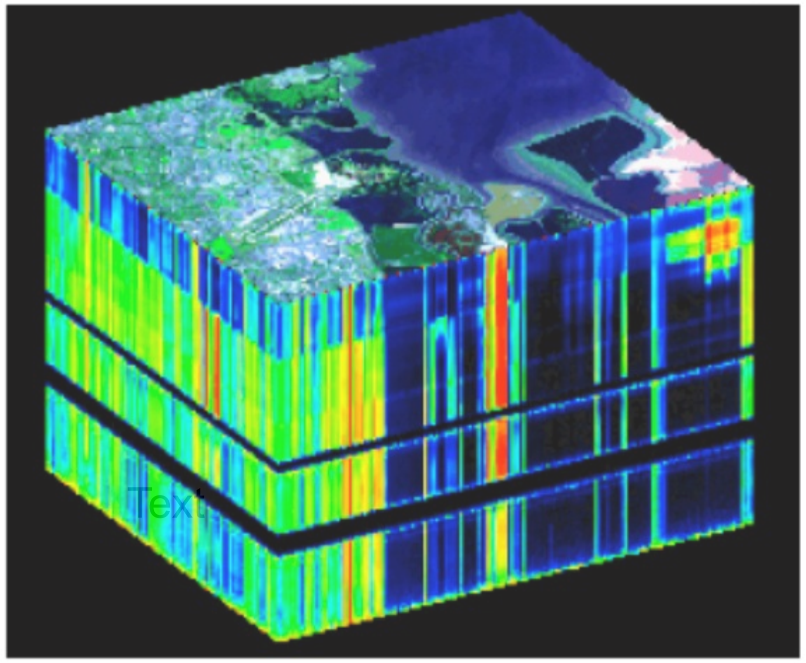


CosmoStat Lab

chan 1 chan 4 chan 7 chan 10

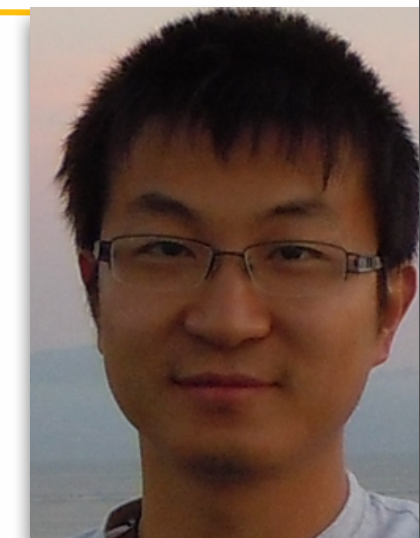
Hyperspectral Sparsity

Marc LENNON
Département Image et Traitement de l'Information (ITI)
École Nationale Supérieure des Télécommunications de Bretagne



$$Y_i = H_i * X_i + N$$

$$X_i = \sum_{s=1}^{S} a_{i,s} E_s \qquad E_s = \Phi \alpha_s$$



$$Y_i = H_i * \sum_{s=1}^{S} a_{i,s} \Phi \alpha_s + N$$

Joint Blind Source Separation and Deconvolution Problem

$$\min_{A, \alpha_s} \parallel Y_i - H_i * \sum_{s=1}^{S} a_{i,s} \Phi \alpha_s \parallel^2$$

=> See Ming's Poster

Conclusions

- ✓ **MRI and Radio-Astronomy Imaging present similar inverse problems**
  - ➡ Sparse Recovery is very efficient
  - ➡ Clear link with Compressed Sensing Theory

- ✓ **Radio-Astronomy**
  - ➡ Improve the resolution by a factor 2.

- ✓ **MRI Imaging**
  - ➡ Divide the acquisition time by a factor 8.

- ✓ **Perspective: CS Hyperspectral Imaging**
  - ➡ **Big computational challenges**
  - ➡ **New sparse modeling**
  - ➡ **New applications (agriculture, medical, etc)**