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Compressed Sensing in MRI: how the Maximum Undersampling Factor depends on the Image Size

C. Lazarus[1], A. Vignaud[2], P. Ciuciu[1]
[1]Gif-sur-Yvette,FR,CEA,DRF/I2BM/NeuroSpin/UNATI/Parietal, [2]Gif-sur-Yvette,FR,CEA,DRF/I2BM/NeuroSpin/UNIRS

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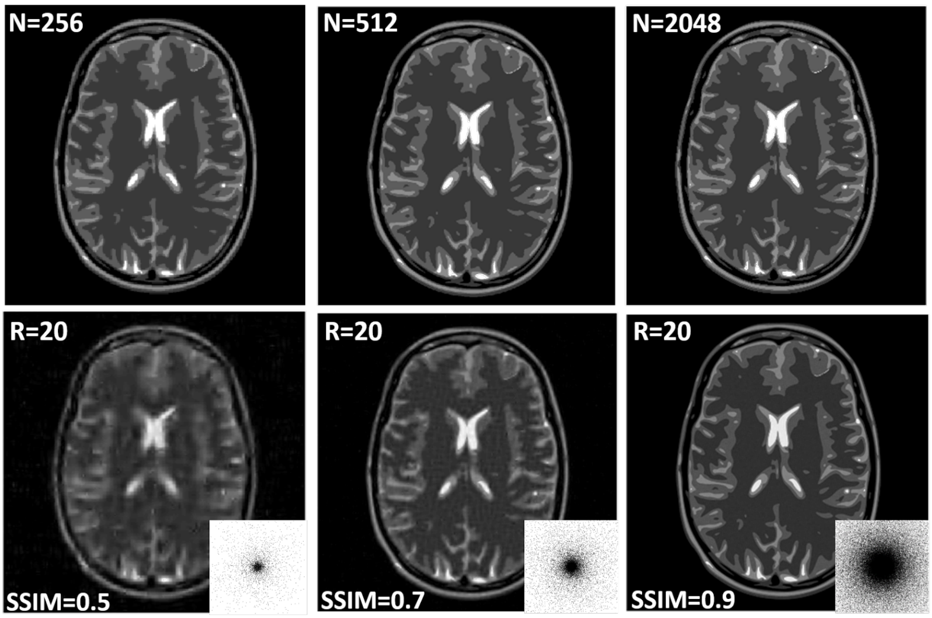
Purpose / Introduction

Since the work of Lustig et al. on Sparse MRI [1], Compressed Sensing (CS) has promised great opportunities to drastically shorten the acquisition time in MRI by reconstructing images from undersampled Fourier data. Although CS theories provide upper bounds relating the number of required measurements m to the image sparsity s and its number of pixels NxN to guarantee exact recovery in the noise-free case ($m \geq 2s \log^2 N$), in practice (noisy case) it remains unclear to which extent MRI acquisitions can be accelerated while preserving image quality. More precisely, finding the relationship linking the maximum achievable undersampling factor $R = N^2/m$ to the image resolution in a noisy context is still an open question. In this numerical study, we propose hints that may guide CS-MRI users in their choice of an appropriate undersampling factor as a function of the image size for different noise levels.

Subjects and Methods

Simulations were performed on a 2D brain image (Fig.1) for increasing image sizes and noise levels characterized by their input SNR (dB). To produce noisy images, complex Gaussian white noise with varying standard deviation σ was added to the Fourier data which was then undersampled by acceleration factors of 5, 10 or 20: samples were randomly picked according to a variable density [2]. Nonlinear non-Cartesian reconstructions were implemented using FISTA algorithm [3] for solving the penalized CS L_1-minimization problem. To assess image quality, we compared the SSIM [4], measuring the similarity in structure with the full k-space image I_0, and the output SNR (dB) calculated as $SNR=10 \log_{10}(||I_0||^2/||I-I_0||^2)$.

Figure 1:

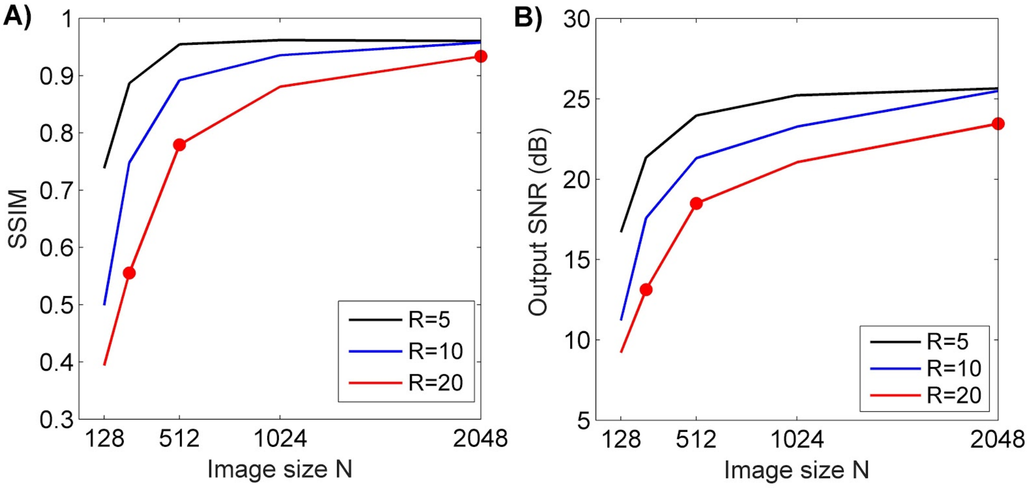


In the noise-free case, examples of fully sampled brain images (top row), and their 20-fold accelerated reconstructions (bottom row) for increasing image sizes. Undersampling Fourier sampling schemes are illustrated in bottom right hand corner of reconstructions. As the image size increases, both SSIM and visual quality are improved. Nonlinear reconstructions involved a redundant wavelet transform taken from RICE toolbox [5] and the NFFT [6] was used in the data consistency term to handle non-Cartesian Fourier samples.

Results

Results for noise-free data are shown in Fig.2 where SSIM and output SNR are represented as a function of image size for the considered acceleration factors. Two regimes can be identified: while image quality is stationary for large image sizes, it rapidly decreases for small decreasing values of N.

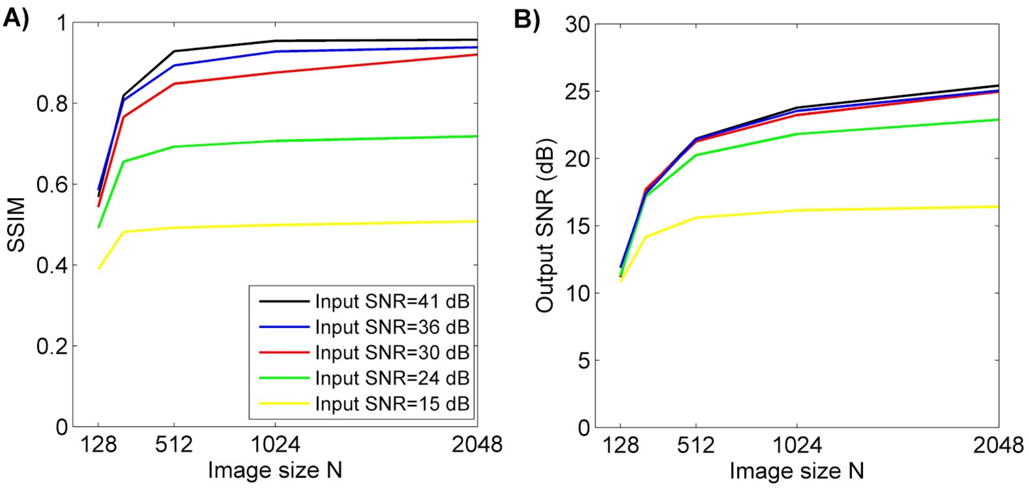
Figure 2:



In the noise-free case, evolution of: A) SSIM and B) output SNR (in dB) as a function of image size N for three acceleration factors R=5 (black), R=10 (blue) and R=20 (red). At a fixed acceleration factor, SSIM and output SNR are both increasing with N, very rapidly for small image sizes and SSIM becomes stationary (close to its maximum value of 1) for large image sizes, especially for R=5-10. Red dots on R=20 curves correspond to the images displayed in Fig.1.

Regarding 10-fold undersampled noisy data (Fig.3), input SNR larger than 24 dB show the same dynamics, maintaining high image quality scores close to the noise-free situation before collapsing at low resolution.

Figure 3:



In the noisy case, evolution of: A) SSIM and B) output SNR (dB) of 10-fold accelerated reconstructed images (R=10 only) as a function of image size N for five increasing noise levels: input SNR of 41 dB (black), 36 dB (blue), 30 dB (red) and 24 dB (green) and 15 dB (yellow). Input SNR (dB) of noisy images was calculated as 20log(signal/noise) where 'signal' refers to the mean signal of a ROI taken in the white matter, 'noise' to the standard deviation in the background signal and log is the decimal logarithm. Here, the reference for computing the SSIM was taken as a very high SNR in standard MRI (SNR=41 dB) while the output SNR was calculated on full k-space noisy images.

Discussion / Conclusion

Noise-free simulations showed how undersampling factors should be chosen according to image sizes: if a threshold of 0.7 in SSIM is considered to give satisfactory image quality (Fig.1 N=512), R should not exceed 5 for N=256 while very large R (≥20) are only achievable for high resolution images of N>1024. Fig.3 suggests that these results can be extended to experimental situations for high enough input SNR.

References

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