DE LA RECHERCHE À L'INDUSTRIE

Compressed Sensing in MRI:

How the maximum undersampling factor depends on the image size and the SNR?

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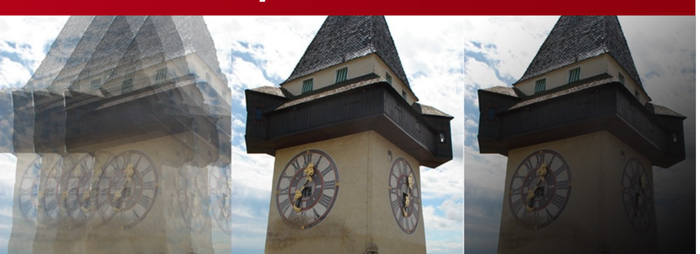
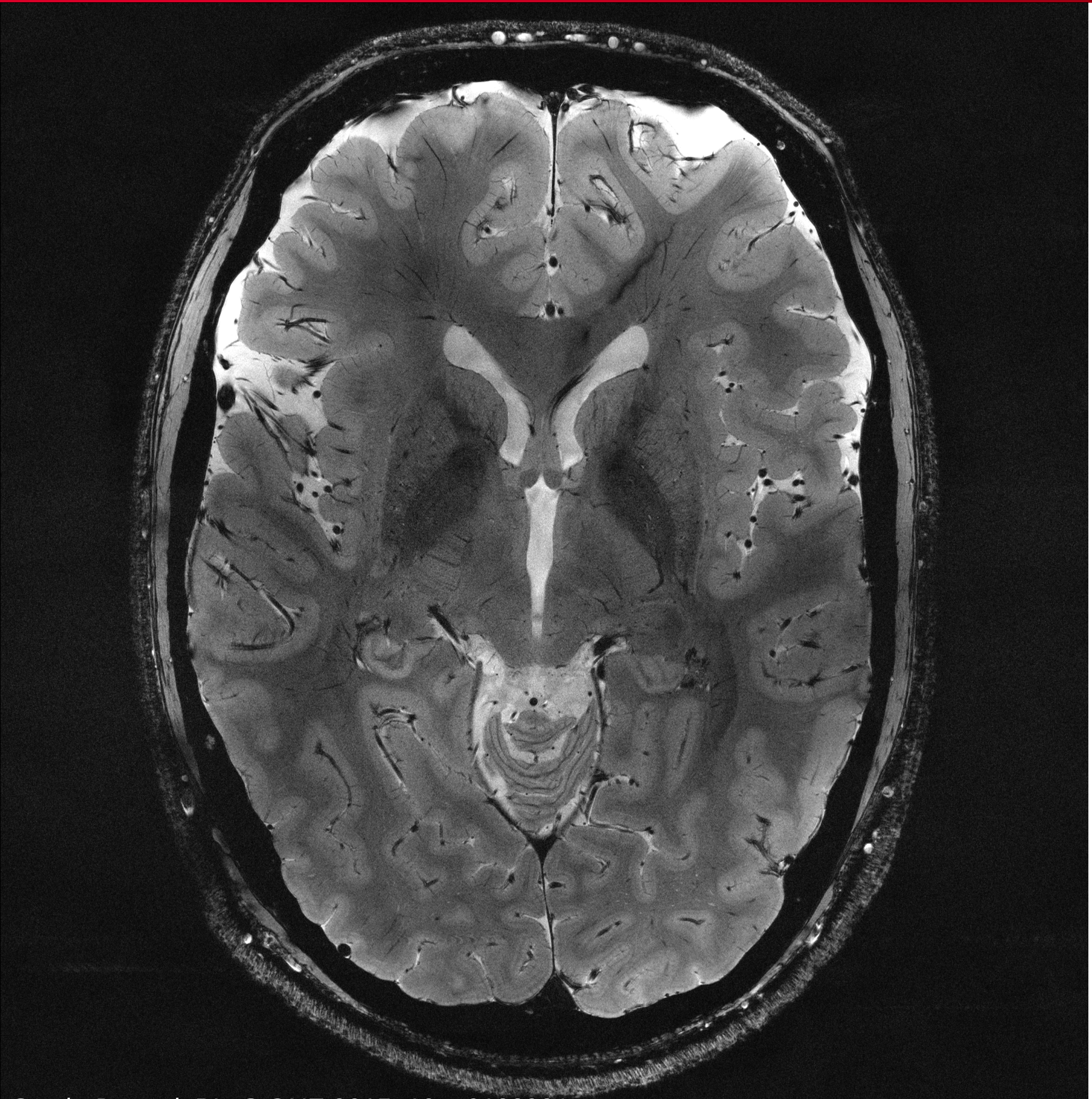
SFB workshop, Graz, Austria
Imaging with Modulated/ Incomplete Data 2016



22-09-2016

INTRODUCTION
HIGH RESOLUTION



Stucht D, et al. PLoS ONE 2015; 10: e0133921

2D T2*w axial, 7T scanner

$120 \times 120 \times 600\ \mu m^3$

Matrix size: 1690 x 1744

21 slices, 2 averages

32-channel receiver coil,

Motion correction

Acquisition Time of 50 minutes!

How can we accelerate this?

INTRODUCTION
QUESTIONS

One solution to reduce the acquisition time:

Undersample the Fourier data using Compressed Sensing theory.

- *Is there a practical user guide to select the appropriate undersampling factor in the particular case of CS-MRI ?*
- *How do we define « appropriate » ?*
- *What are the factors influencing this choice?*

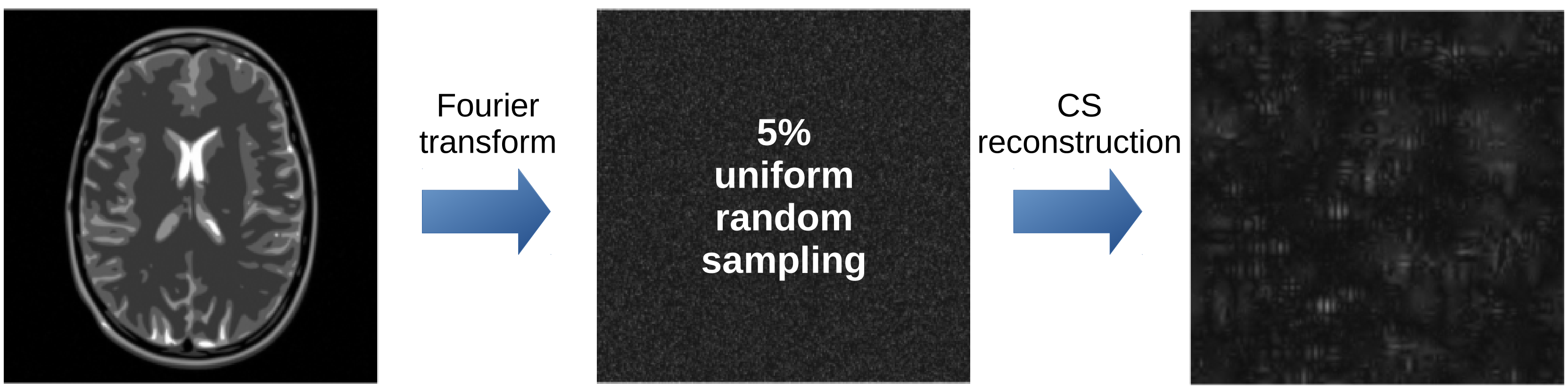
INTRODUCTION
BACKGROUND

- ***From the traditionnal CS theory...***

  ➔ To faithfully recover a signal with s on-zero entries:

  number of required measurements: $m = O(s \cdot log(n))$ where n=#pixels

  ➔ Noisy case: still holds but a larger error



- ***From the traditionnal CS theory...***

  ➔ To faithfully recover a signal with s on-zero entries:

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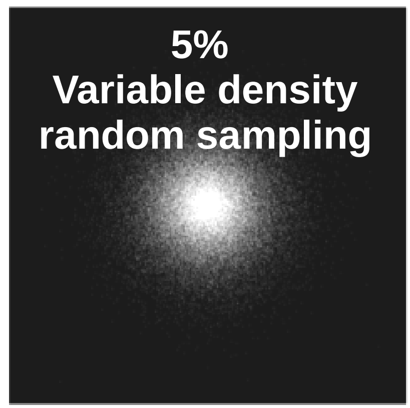
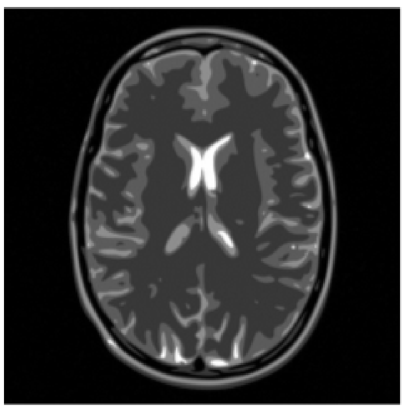


CS
reconstruction



- ***… to a CS adapted to MRI***

  - **CSMRI** [Lustig et al. 2008]
  - **V**ariable **D**ensity **S**ampling [Puy et al. 2011]
  - *Breaking the coherence barrier: A new theory for Compressed Sensing* [Adcock et al. 2013]

« The success of compressed sensing is resolution dependent »

What is the maximum degree of acceleration that can be applied in a given situation?

1. *The larger the image size, the more you can undersample.*
2. *The larger the input SNR, the more you can undersample.*

Our objectives:

- We want to verify these intuitions with simulations and experiments.
- We want to offer quantitative guidance to choose a proper undersampling factor as a function of image size and SNR and desired image quality.

OVERVIEW

I. Materials and methods

- Simulations
- Experiments

II. Results

- Resolution dependence
- SNR dependence
- $R_{max}(\varepsilon, N, SNR)$
- MR feasible sampling schemes

III. Discussion and conclusion

MATERIALS and METHODS

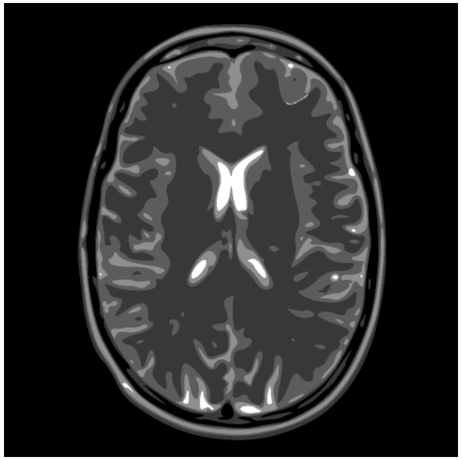
Simulations

II. MATERIALS AND METHODS
A. SIMULATIONS - IMAGES

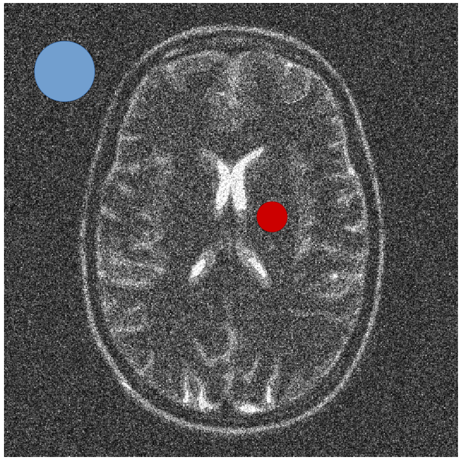
- Analytical phantom of T2-like brain image (infinite SNR) [Guerquin-Kern et al. (2012)]

  Matrix sizes: *N = 128, 256, 512, 1024 or 2048*

  Number of pixels: $n=N^2$



- Addition of complex-valued Gaussian noise, with growing standard deviation, to the Fourier data



- Calculation of the input SNR on magnitude image:

  SNR = S / σ

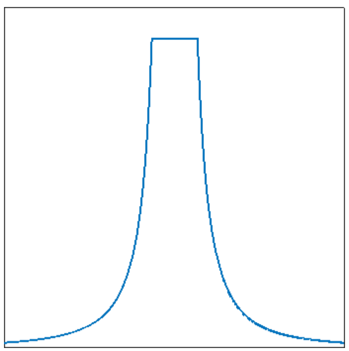
  - S = mean signal in a ROI in white matter
  - σ = standard deviation in the background

  ➔ SNR ranges from 6 to 110

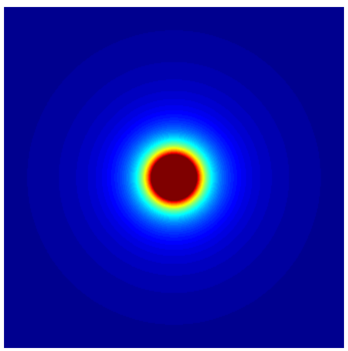
II. MATERIALS AND METHODS
SIMULATIONS - UNDERSAMPLING

- Retrospective undersampling factors $R=N^2/m$ ranging from 2 to 30
- Variable Density Sampling with a polynomial decay of $1/|k|^2$

Not MR feasible but provides near upper bounds to sampling performances



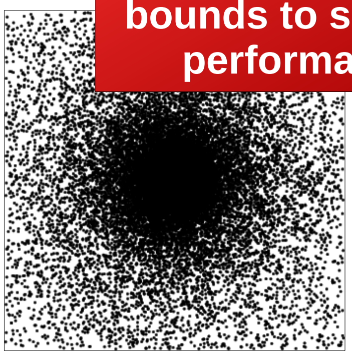
Radial view of density



2D view of density

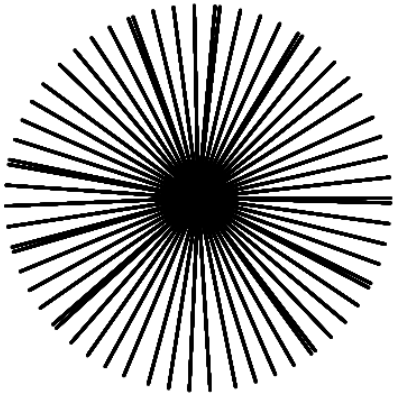
Non-Cartesian

iid sampling



Samples in k-space
N=512 and R=5

- Example of **MR feasible sampling schemes**
  - Radial golden angle [Winkelmann et al. (2007)]



II. MATERIALS AND METHODS
SIMULATIONS - RECONSTRUCTION

- L2-L1 penalized CS problem:

$$\underset{z}{\text{minimize}} \overbrace{\|Az - y\|_2^2}^{\textit{Data consistency}} + \underbrace{\lambda \|z\|_1}_{\textit{Enforces sparsity}}$$

  - NFFT [Keiner, Kunis and Potts, TU Chemnitz]
  - Redundant wavelets from RICE university toolbox
  - Constant regularization parameter $\lambda=10^{-4}$

- FISTA algorithm [Beck & Teboulle (2009)]

II. MATERIALS AND METHODS
SIMULATIONS – IMAGE QUALITY METRIC

- **SSIM: The Structural SIMilarity (SSIM) index [Wang et al. (2004)]**
  - Measures the similarity between 2 images.
  - *SSIM(I, Iref) = 0* : null correspondence.
  - *SSIM(I, Iref) = 1* : perfect match.
  - A measure closer to the human perception of error in an image than traditional error metrics such as MSE or pSNR.
  - **MR main customer: physicians and radiologist.**

- **Our choice of reference**:

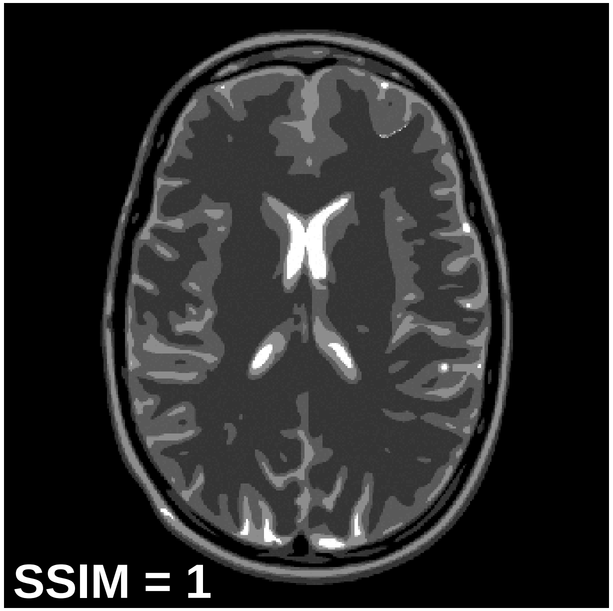
  The corresponding full Cartesian image with a high input SNR of 110.

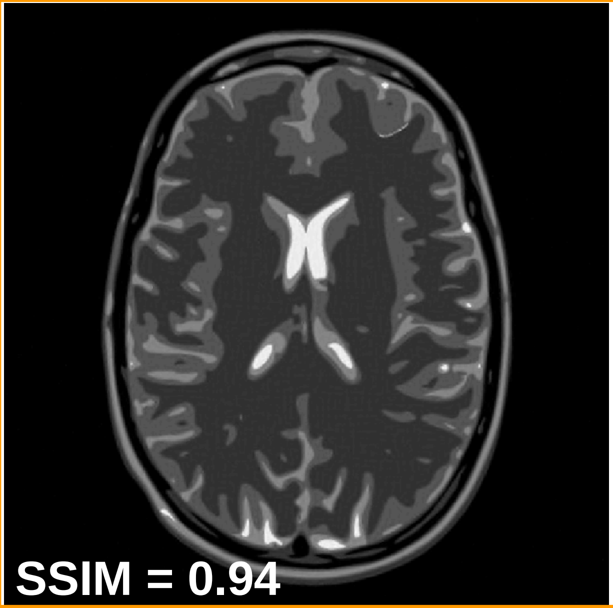
- ***What SSIM threshold?***

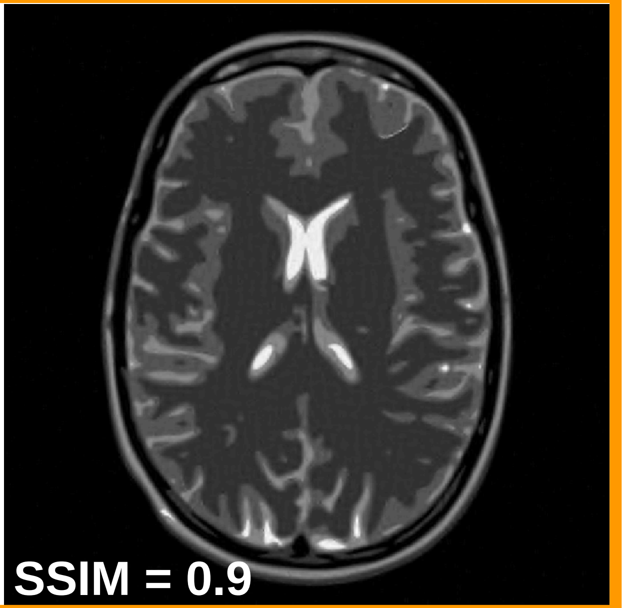
II. MATERIALS AND METHODS
SIMULATIONS – IMAGE QUALITY THRESHOLD

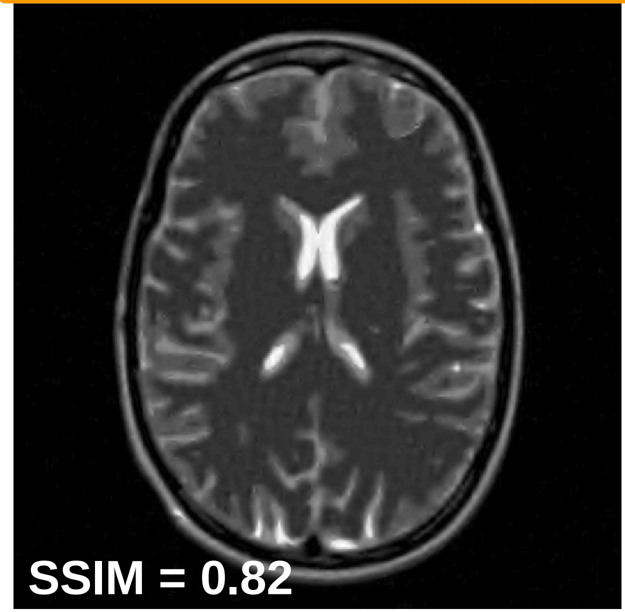
N = 512

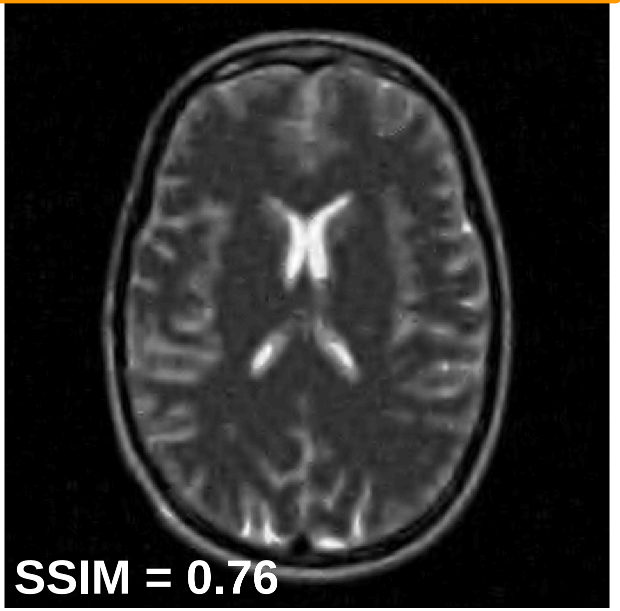
Reference











We choose an image quality threshold of:

$SSIM \geq 0.9$

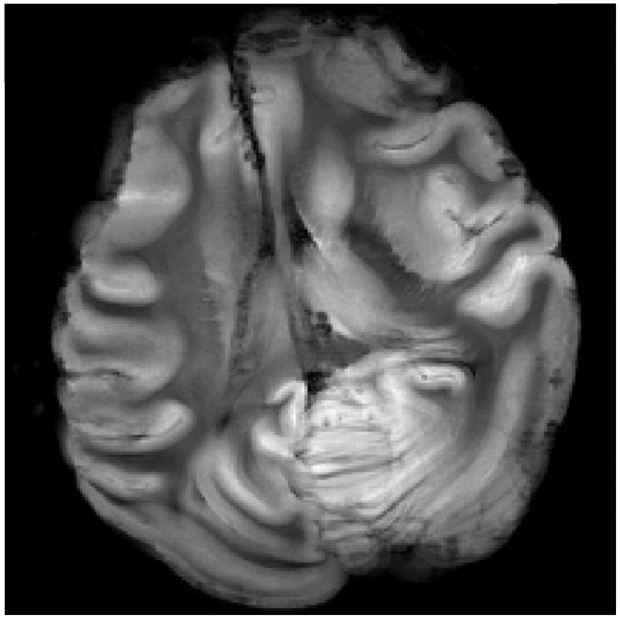
MATERIALS and METHODS

Experiments

II. MATERIALS AND METHODS
B. EXPERIMENTS

- *Ex-vivo* brain baboon in fluorinert solution

- **Sequence parameters**
  - 7T Siemens healthineers scanner
  - In vivo Corp Birdcage 1Tx/1Rx coil
  - T2* weigthing
  - Slice thickness: 5 mm
  - TR = 60 ms
  - TE = 30 ms
  - α = 10°
  - Axial slice



Full Cartesian acquisition N=512 of SNR = 110

- Signal averaging to increase *input SNR*

- Retrospective undersampling with the same samples as for simulated images

OVERVIEW

I. Materials and methods

- Simulations
- Experiments

II. Results

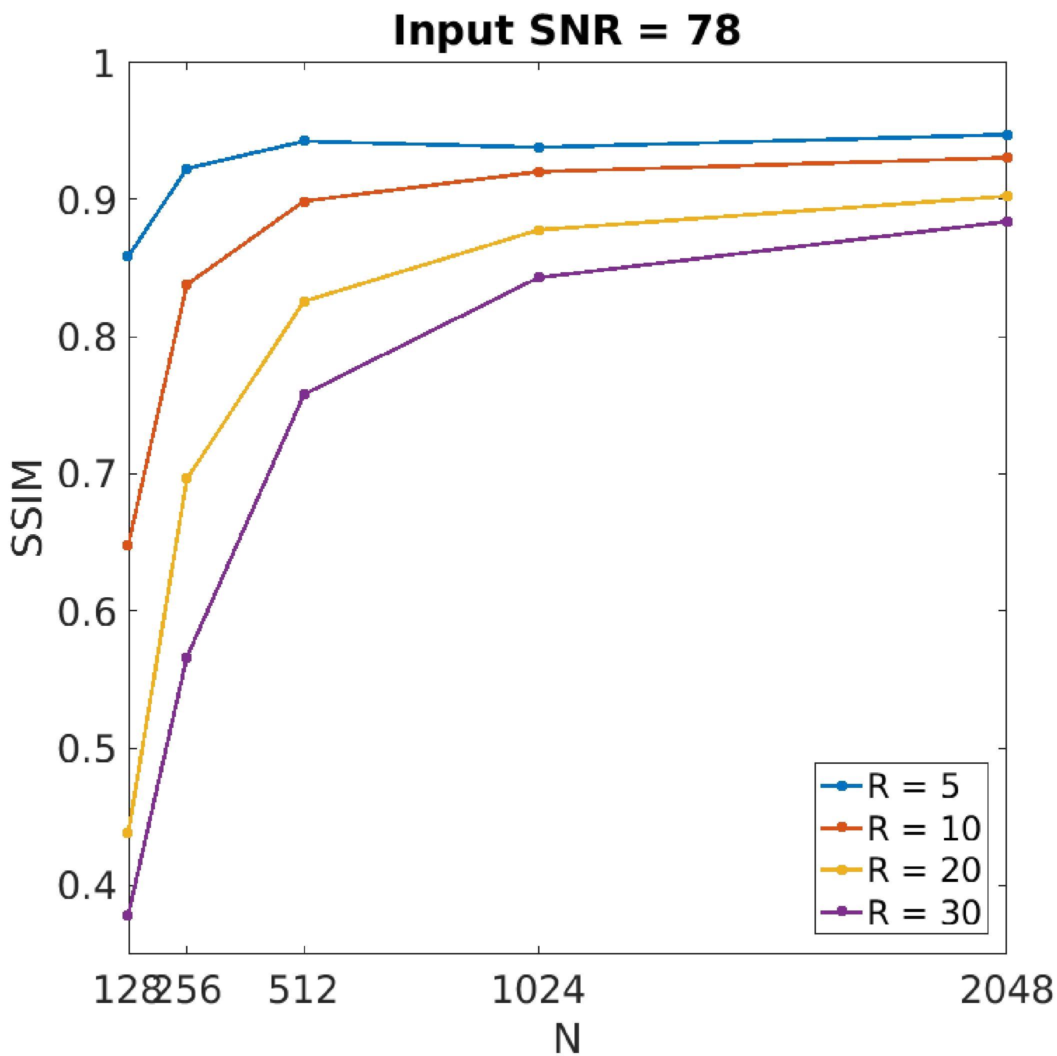
- Resolution dependence
- SNR dependence
- $R_{max}(N, SNR)$
- MR feasible sampling schemes

III. Discussion and conclusion

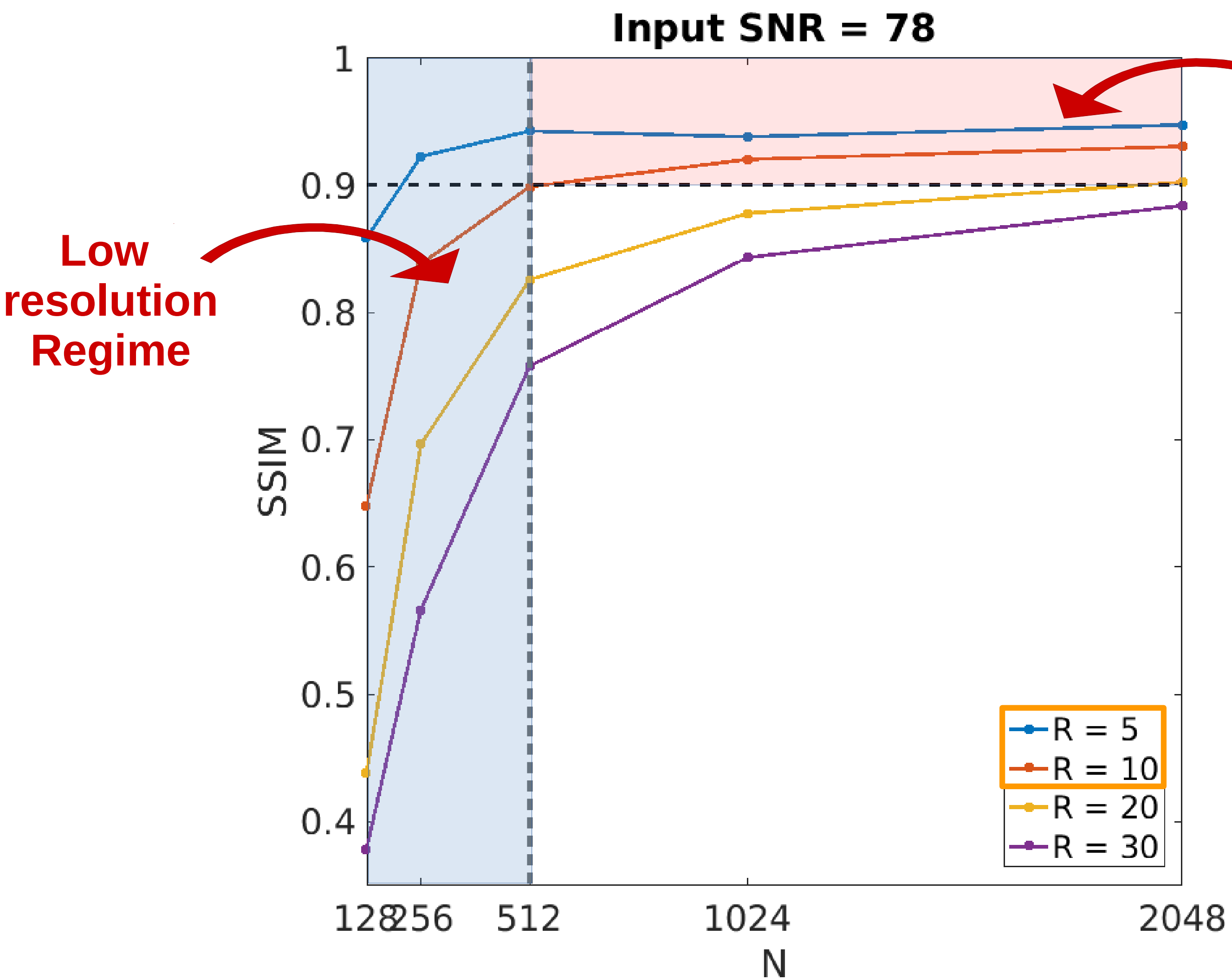
Resolution dependence

How should the acceleration factor be chosen as a function of image size ?

RESOLUTION DEPENDENCE

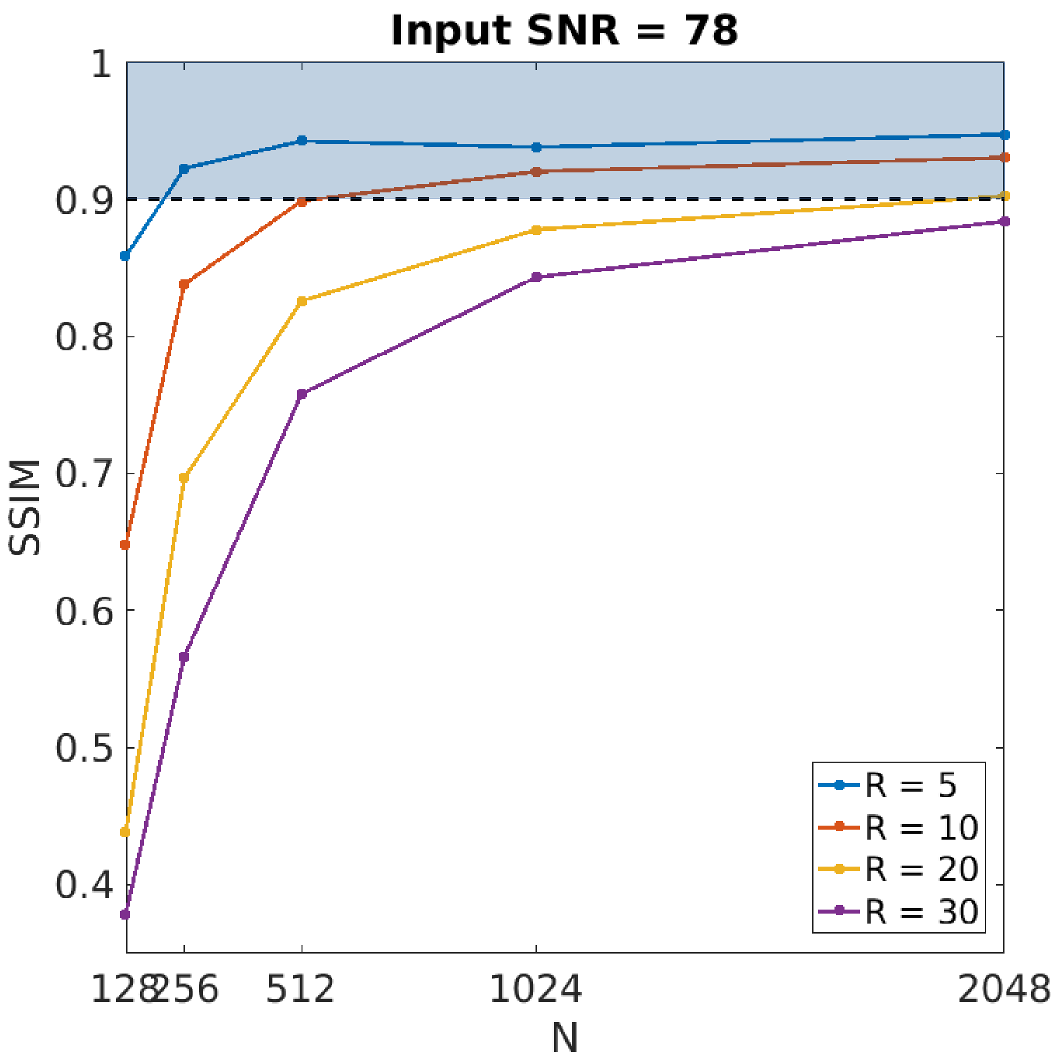


RESOLUTION DEPENDENCE



What about the dynamics of SSIM evolution?

RESOLUTION DEPENDENCE



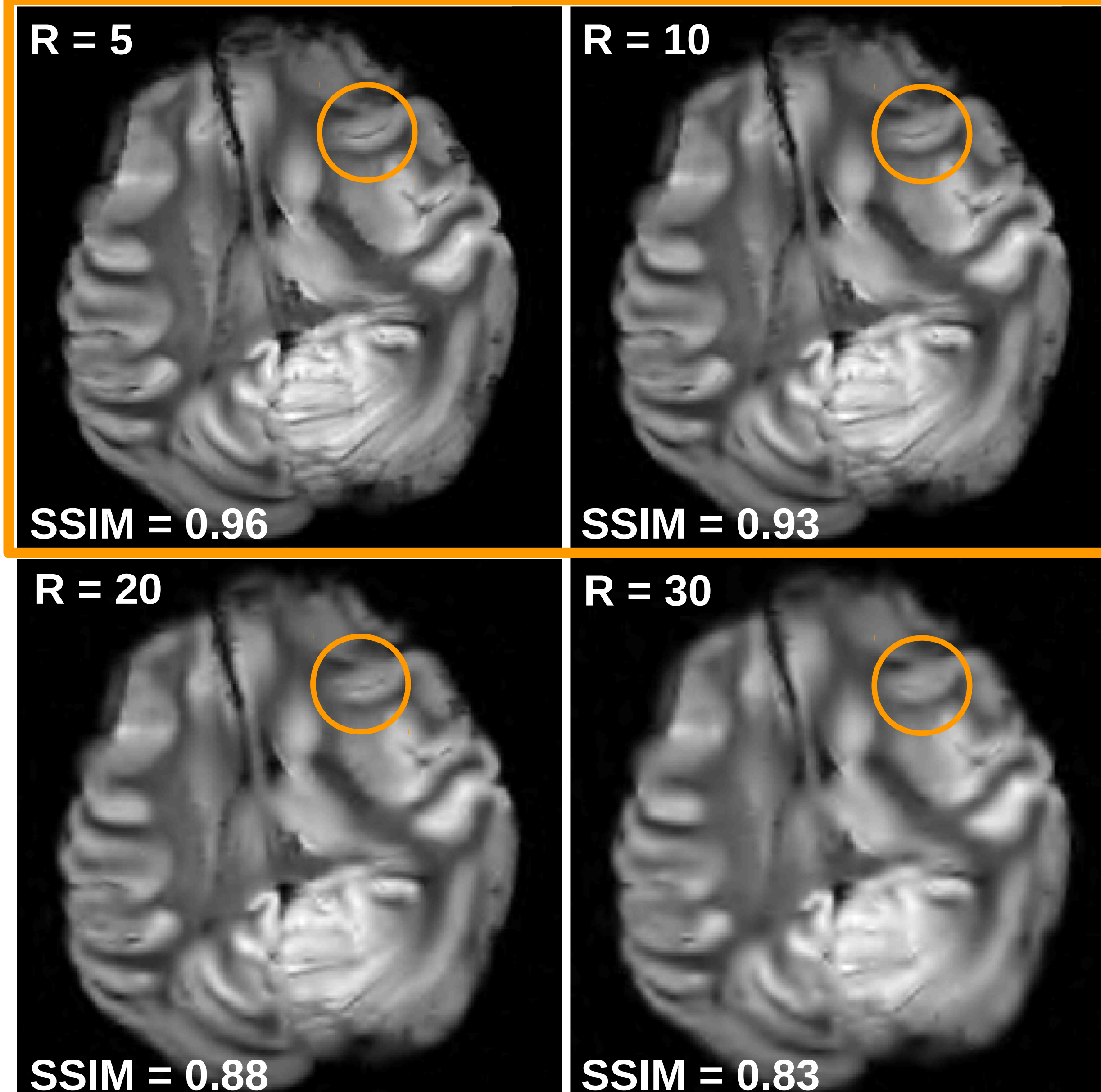
- There is a restricted number of authorized combinations:
  - R=5 works for N≥256
  - R=10 works for N≥512
  - R=20 works for N≥2048
  - ...
- *Do these figures make sense on experimental data?*

RESOLUTION DEPENDENCE EXPERIMENTS - VISUALIZATION

N = 512

Input SNR = 74

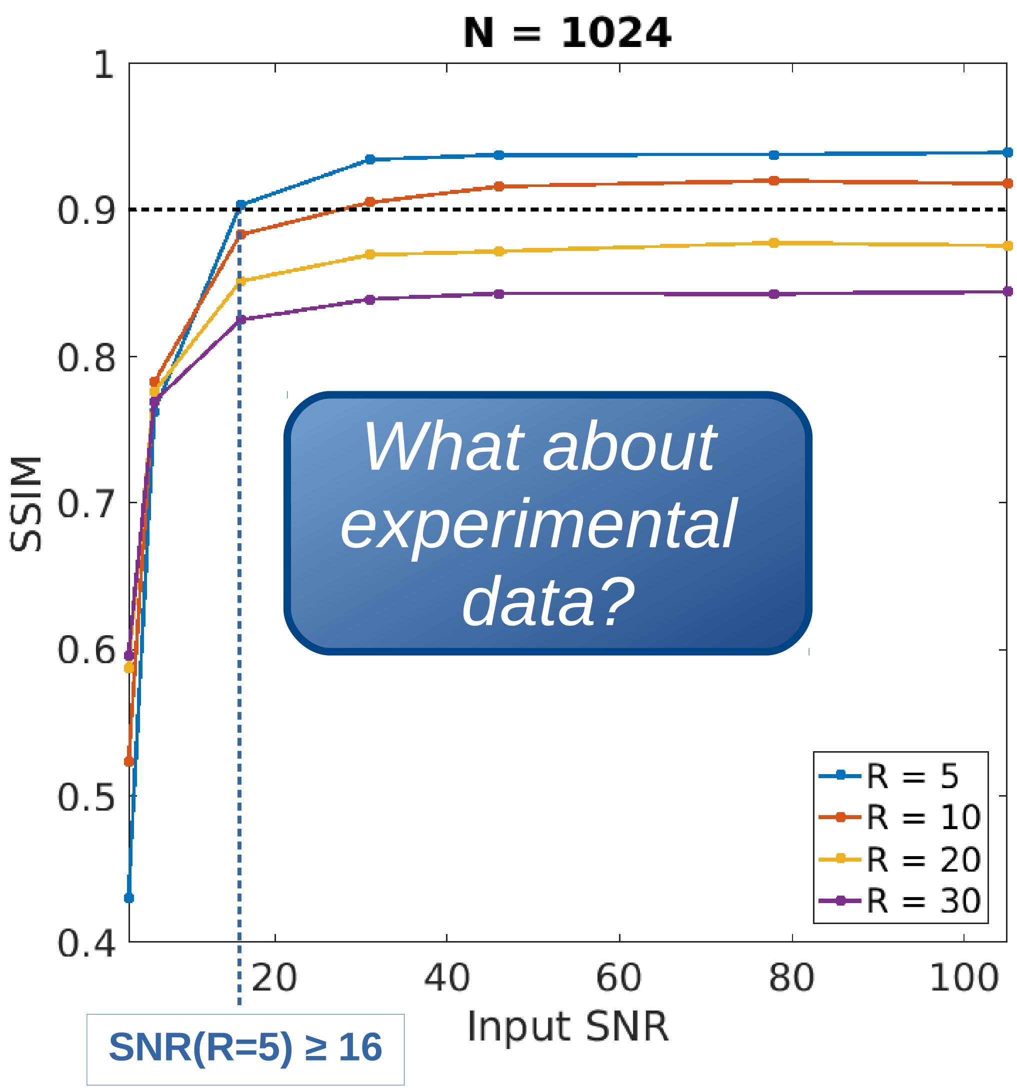
Reference



SNR dependence

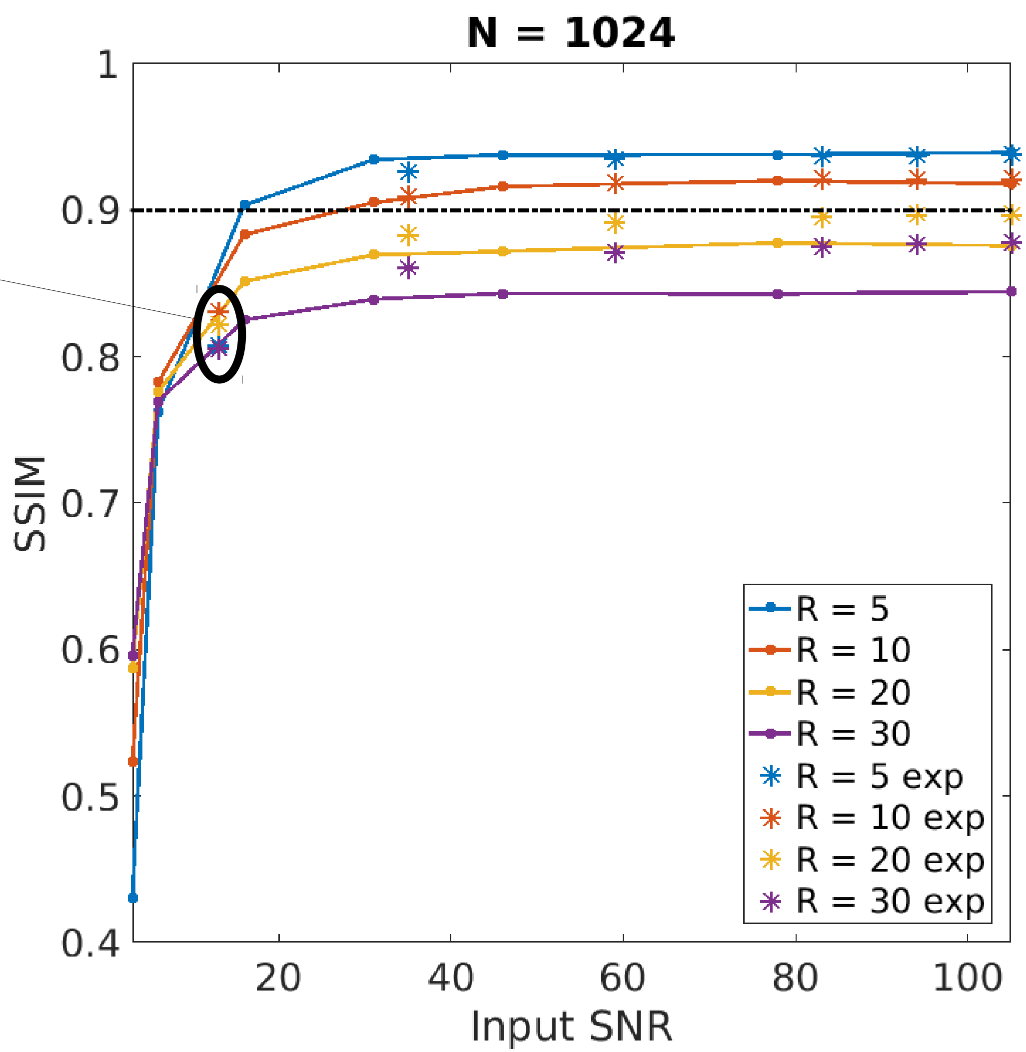
How should the acceleration factor be chosen as a function of SNR?

SNR DEPENDENCE
IDEA OF A MINIMUM INPUT SNR(R,N)



SNR DEPENDENCE
EXPERIMENTAL POINTS

Only one average



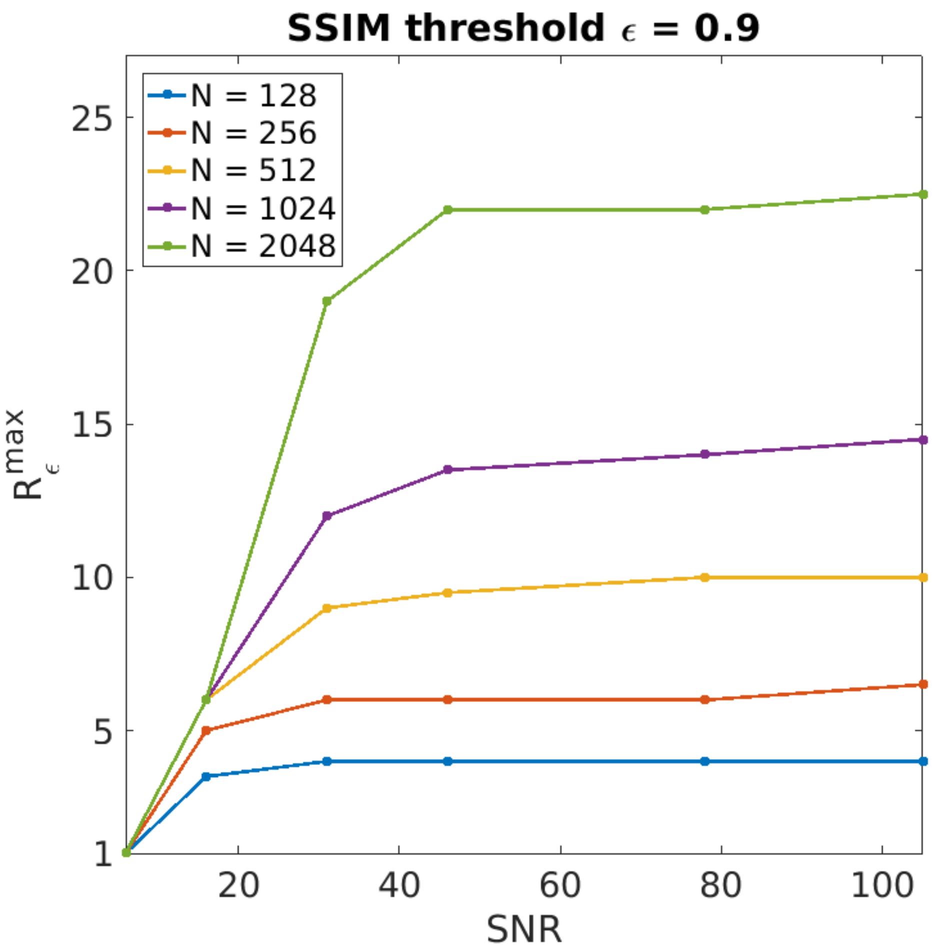
II. RESULTS
$R_{MAX}(E, N, SNR)$

$$R_{max}(\varepsilon, N, SNR)$$

Towards a maximum undersampling factor dependent on image quality threshold ε, image size N and *input SNR*?

II. RESULTS
$R_{MAX}(\varepsilon, N, SNR)$

We introduce: $R_{max}(\varepsilon, N, SNR) = \max \{ R: SSIM(I\,;\,I_{ref}) \geq \varepsilon \}$

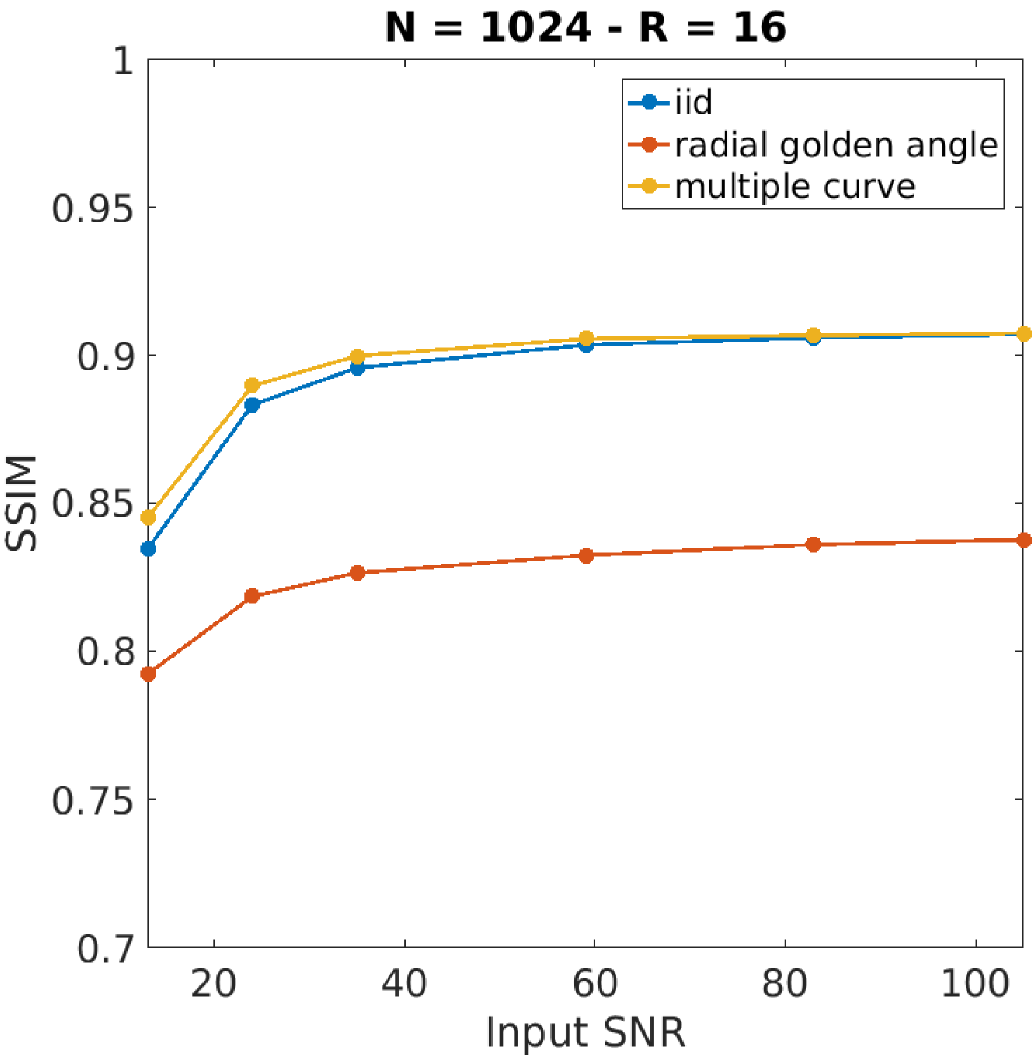


MR feasible sampling schemes

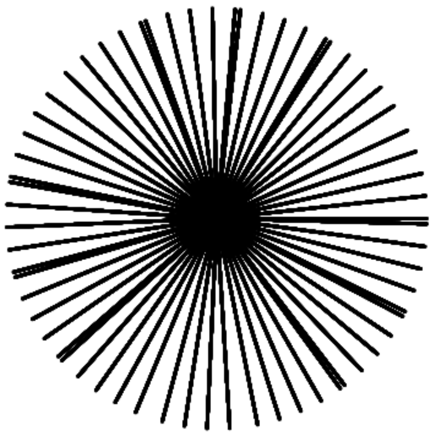
Radial golden angle
Multiple curve sampling

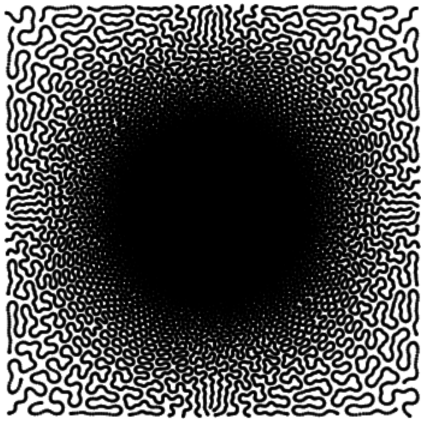
How do they perform compared to pointwise samples?
Can we extend results on iid sampling to MR feasible sampling?

II. RESULTS
MR FEASIBLE SAMPLING SCHEMES



Radial golden angle - R=16





Multiple curve - R=16
[Chauffert et al. (2015)]
[Boyer et al. (2016)]

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III. DISCUSSION AND CONCLUSION

Compressed Senging in MRI:

What is the maximum degree of acceleration that can be applied in a given situation?

1) **About resolution dependence**
   - High resolution → Large subsampling factor
   - Quantitative verification
   - This tendency is expected to be even stronger for 3D imaging.

2) **About SNR impact**
   - SNR_{min} to use a given *R* while preserving desired quality.
   - Low SNR occur in high resolution imaging (eg: N=1024)
   - High resolution CS-MRI needs to maximize the input SNR:
     - Ultra High Field in MRI
     - multiple receiver coils.

III. DISCUSSION AND CONCLUSION

3) ***R_{max}(ε, N, SNR)* as a compressed summary**

4) **Results on iid sampling can be extented to MR feasible sampling schemes**

Iid sampling gives an near upper bound to undersampling performances.

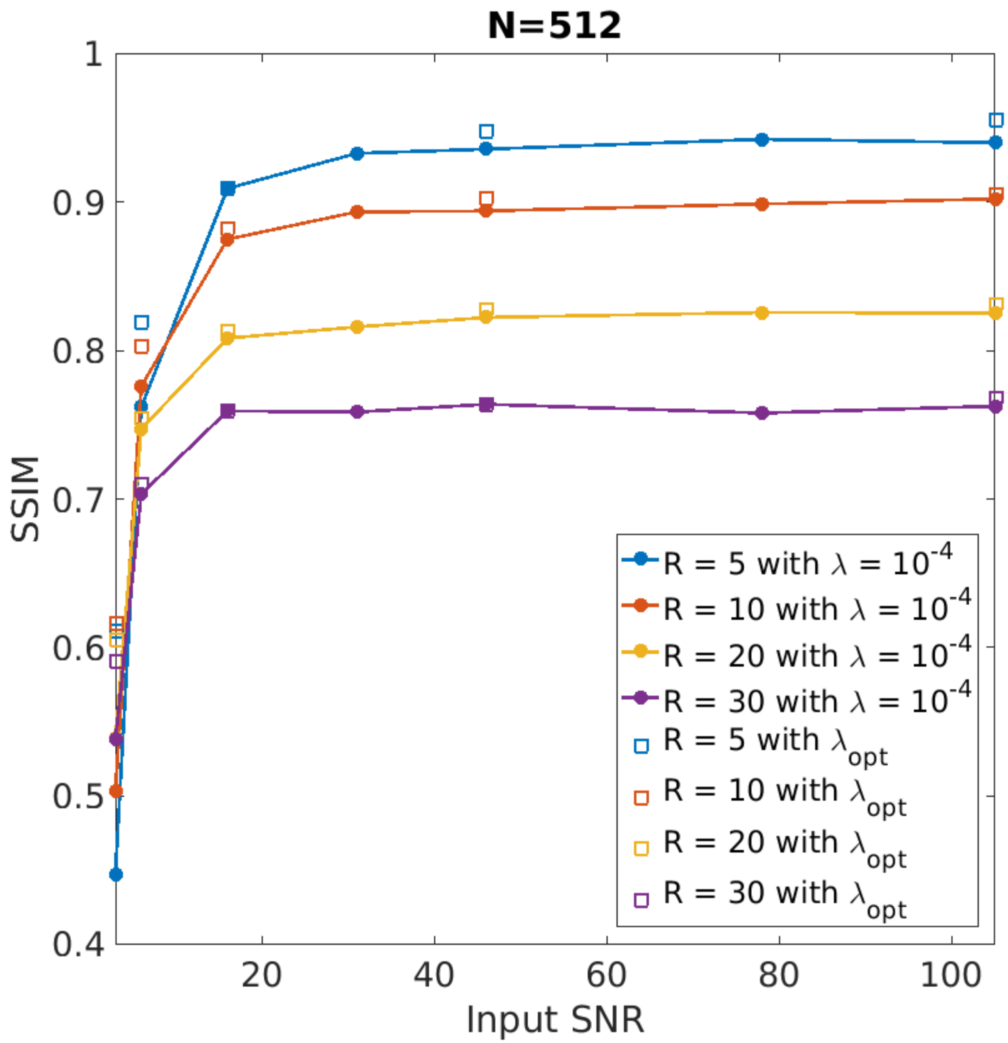
✗ **Some limitations of the study:**

- Reconstruction
  - Other reconstructions may lead to higher image quality
- Image quality metrics
- Results could be extented to prospective CS-MRI

Thanks for your attention!

Any questions?

APPENDIX
EFFECT OF REGULARIZATION PARAMETER



ACKNOWLEDGMENTS

- We wish to acknowledge Michel Bottlaender for the use of the *ex-vivo* baboon brain.