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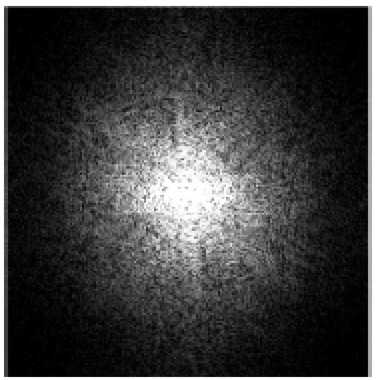
SYNTHESIS VS ANALYSIS SPARSITY PROMOTING MR IMAGE RECONSTRUCTION FROM INCOMPLETE DATA

March 17th, 2017

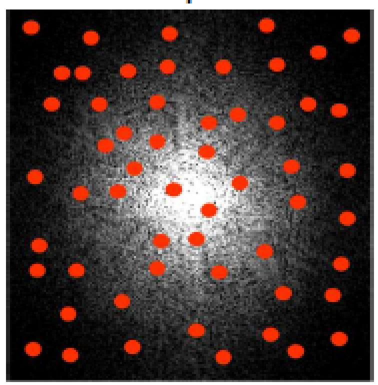
MODELING, PROBLEM STATEMENT

- MRI: Optimize **image reconstruction**

$F^*\Phi\alpha = H\alpha$



$H_\Gamma\alpha$



Observation model: $y = H_\Gamma\alpha + b$

Basic formulation :

$$\hat{\alpha} = argmin\left(f\left(H_\Gamma\alpha\right) + \lambda\Psi(\alpha)\right)$$
$$\hat{\alpha} = argmin\|H_\Gamma\alpha - y\|_2^2 + \lambda\Psi(\alpha)$$

Where :

Ψ Enforces sparsity

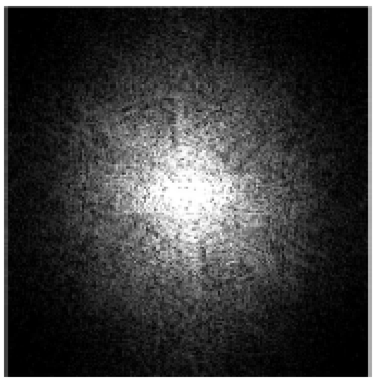
Typical expression :

$$\Psi = \|.\|_1, \|.\|_0 \dots$$

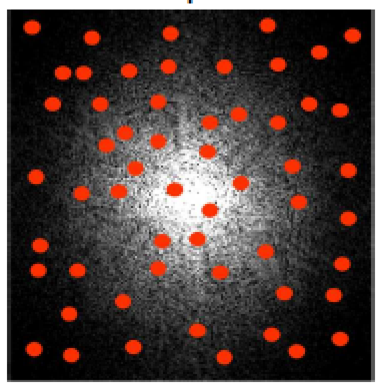
MODELING, PROBLEM STATEMENT

- MRI: Optimize **image reconstruction**

$F^*\Phi\alpha = H\alpha$



$H_\Gamma\alpha$



Observation model: $y = H_\Gamma\alpha + b$

- Synthesis formulation (eg, FISTA):

$$\hat{\alpha} = \underset{\alpha \in \mathbb{C}^p}{argmin} \| y - H_\Gamma \alpha \|_2^2 + \lambda \| \alpha \|_1$$

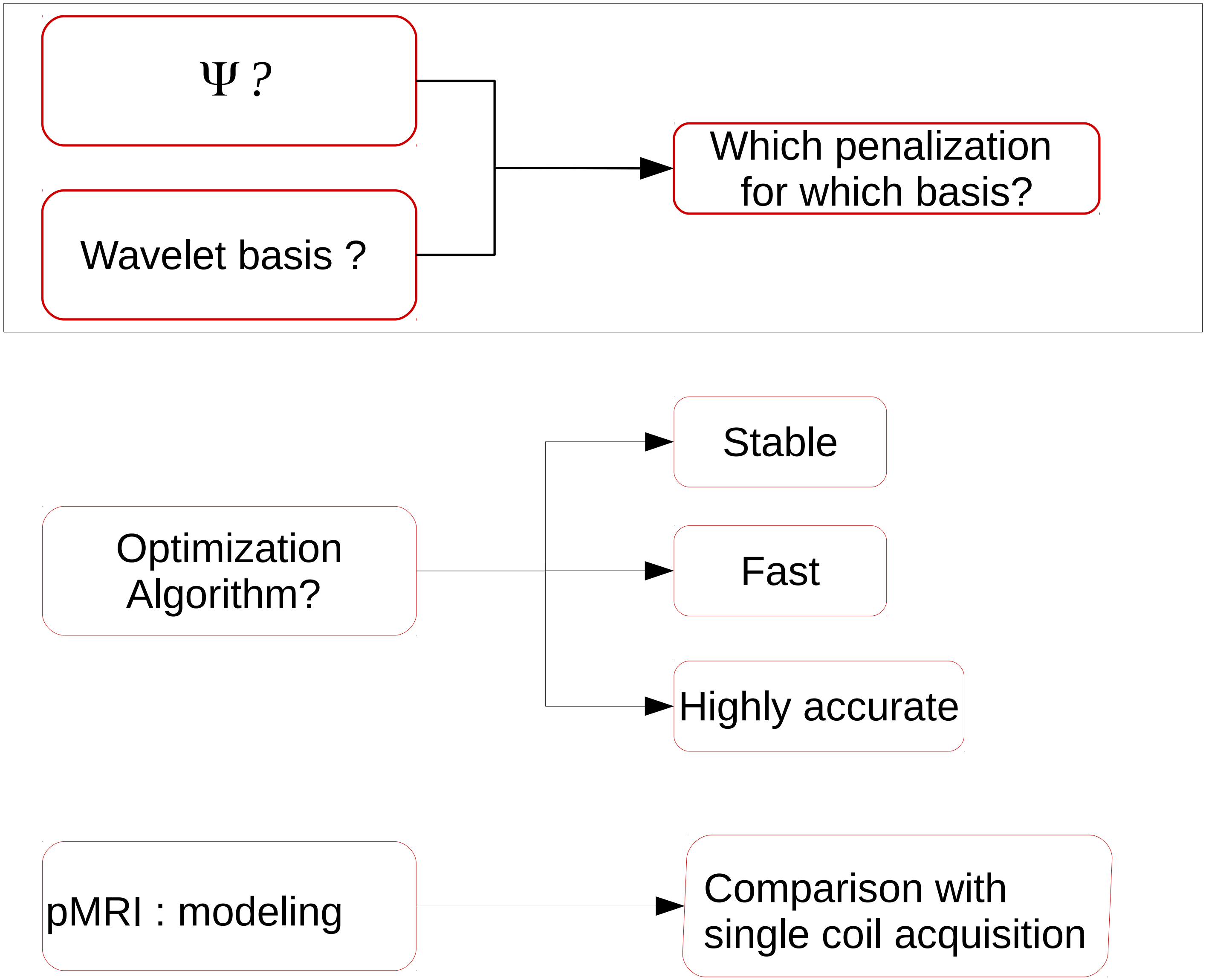
- Analysis formulation (eg, 3MG):

$$\hat{x} = \underset{x \in \mathbb{C}^p}{argmin} \| y - \Gamma F^{¿*¿} x \|_2^2 + \lambda \| \Phi^{¿*¿} x \|_1$$

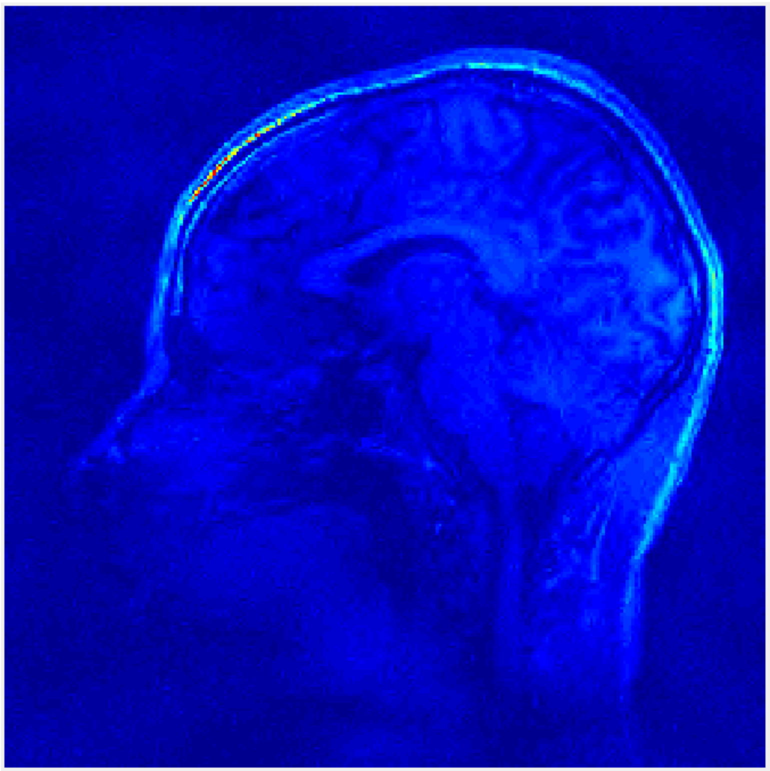
- Equivalence between synthesis & analysis formulations for **wavelet bases**

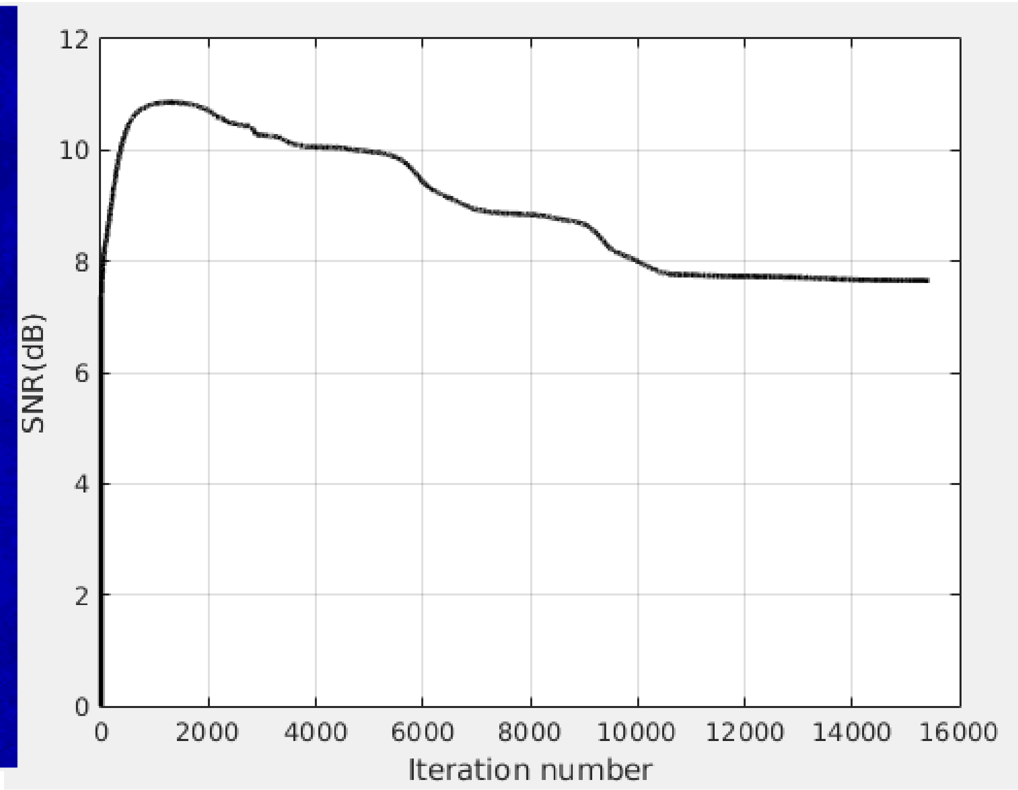
- Extensions to multi-channel data & 3D imaging

RECONSTRUCTION PROBLEM OVERVIEW



STRANGE BEHAVIOR OF THE 3MG ALGORITHM





Output image of the 3MG

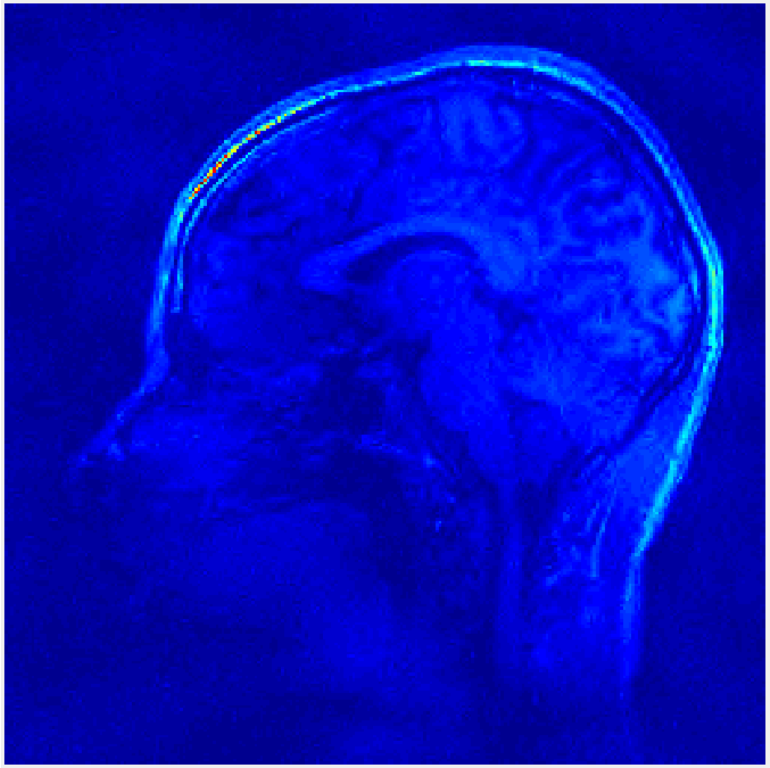
Evolution of the SNR during the minimization process

Why do we have this output ?

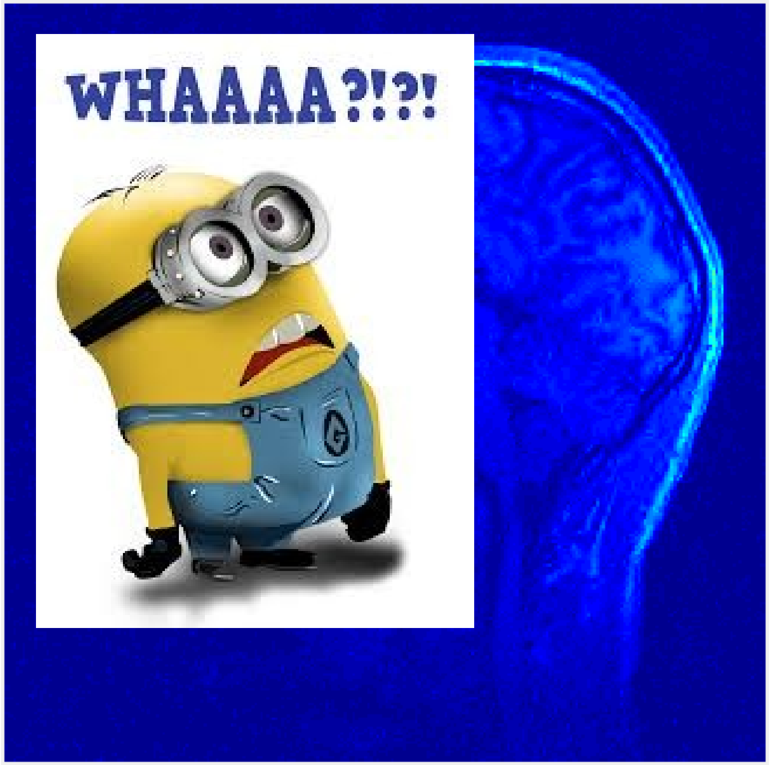
WAVELET TRANSFORM & PENALIZATION

Penalizing all coefficients: details + approximation

- Test : 3MG + Symmlet



Without penalization



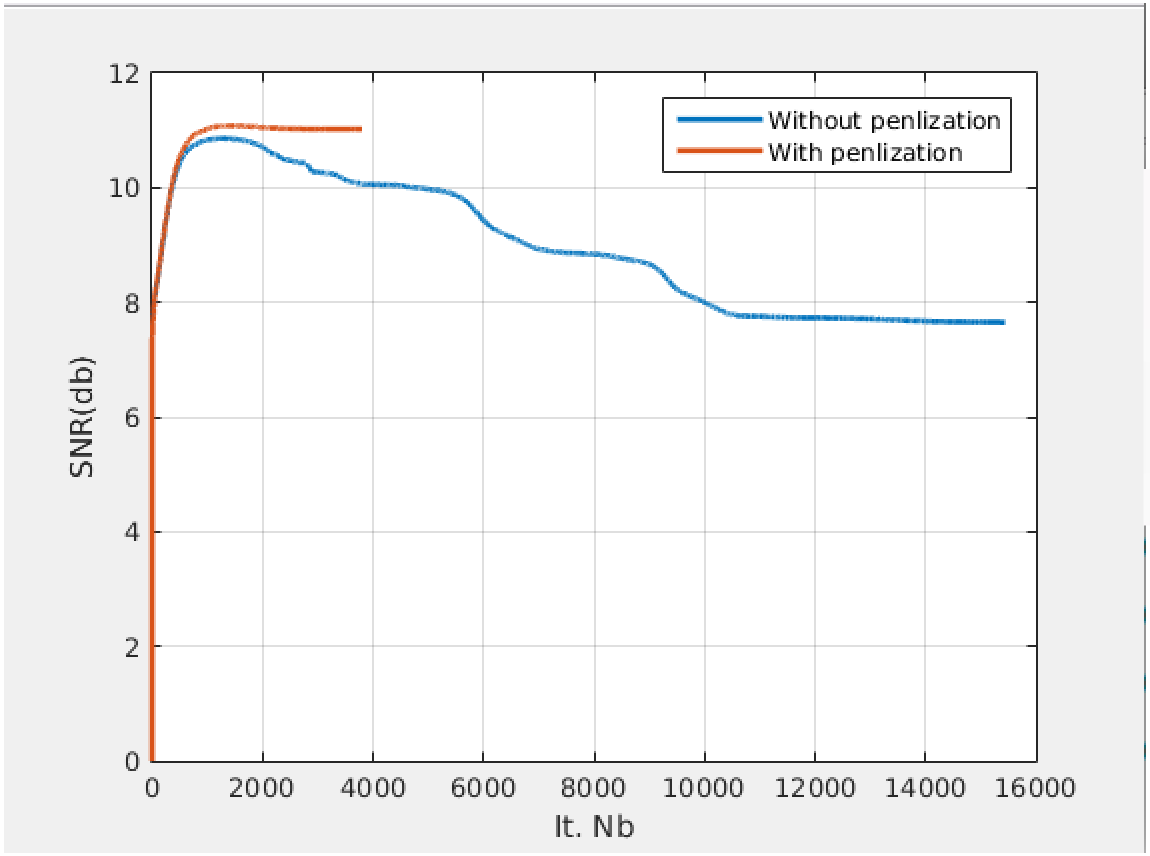
With penalization

WAVELET TRANSFORM & PENALIZATION

Penalization of all the coefficients ?

- Test : 3MG + Symmlet

What happens if we penalize the approximation coefficients?



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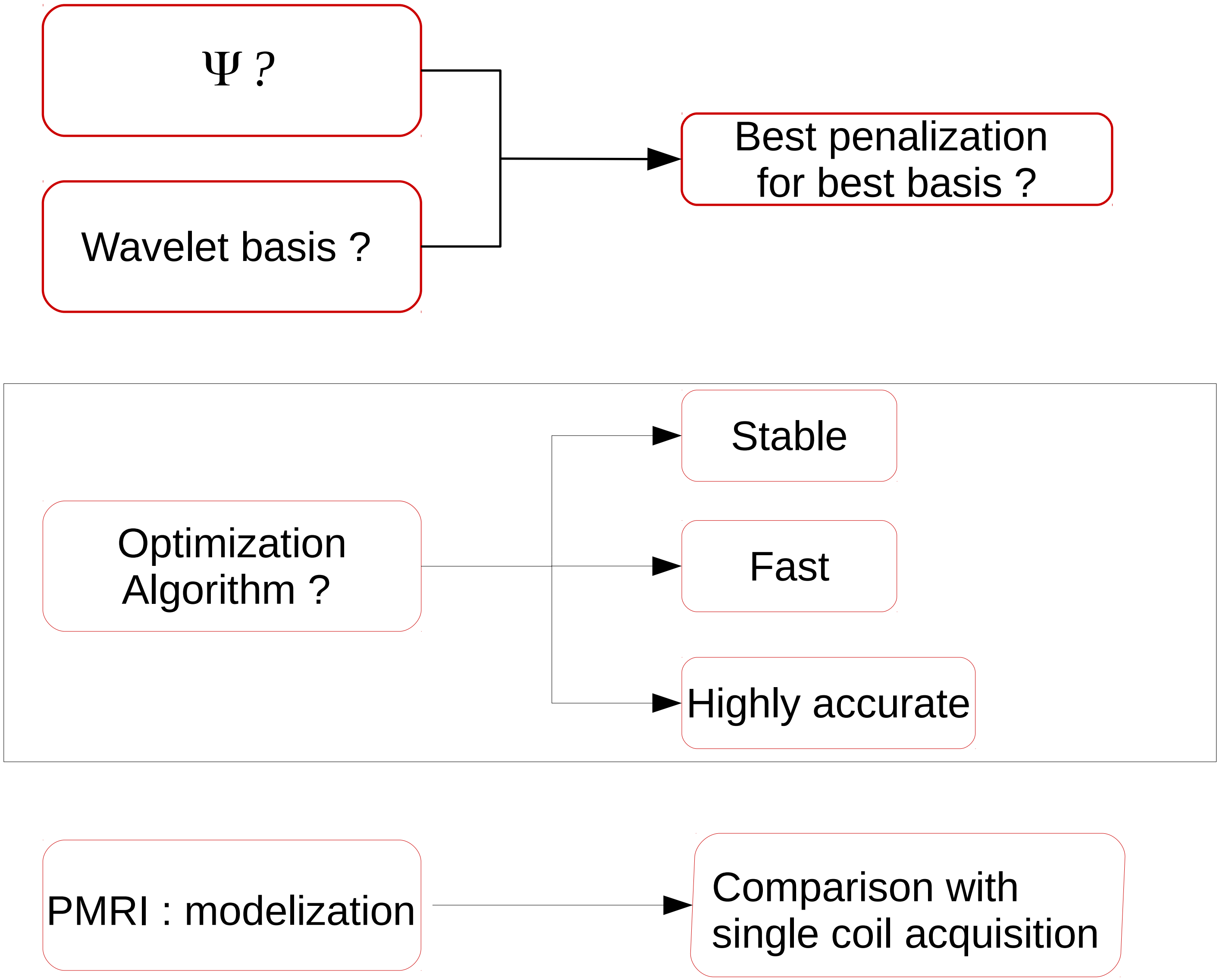
CONCLUSION

The way we penalize the coefficients is as important as the sparse decomposition we are using

RECONSTRUCTION PROBLEM OVERVIEW



EXISTING ALGORITHMS

- FISTA (Fast Iterative Shrinkage Thresholding Algorithm)
  - Synthesis formulation
  - Compatible with the use of the NFFT
  - Single coil acquisition
- 3MG (Majorize-Minimize Memory Gradient Algorithm)
  - Analysis formulation
  - Compatible with the multi-channel acquisition
  - Only implemented with FFT

OVERVIEW OF MY CONTRIBUTIONS

- FISTA
  - Implement the multi-channel reconstruction
- 3MG
  - Implement the non-Cartesian sampling (NFFT)
- Make a comparison between these two algorithms
- Compare different penalizations on various sampling schemes

OVERVIEW OF MY CONTRIBUTIONS

- FISTA (Fast Iterative Shrinkage-Thresholding Algorithm)

First order method:

Based on a proximal approach:

Original implementation using the L_1 norm

- 3MG (Majorize-Minimize Memory Gradient Algorithm)

Based on the minimization of a surrogate function

L_2-L_0 penalization function:

$$\Psi(u)=\frac{u^2}{2x\delta^2+u^2}$$

L_2-L_1 penalization function:

$$\Psi(u)=\sqrt{\frac{u^2}{\delta^2}+1}-1$$

First order method:

For *f* convex and gradient-Lipschitz function we can easily find a majorizing function

$$f(x) \leqslant f(y) + \nabla f(y)^T (x-y) + \frac{L}{2}\|x-y\|_2^2$$

$$\hat{\alpha} \in argmin\, f(\alpha) + \|\alpha\|_1 \Rightarrow \hat{\alpha} \in argmin_\alpha f(\alpha') + \nabla f(\alpha')^T (\alpha - \alpha') + \frac{L}{2}\|\alpha - \alpha'\|_2^2 + \|\alpha\|_1$$

$$\hat{\alpha} \in argmin_\alpha \frac{L}{2}\left\|\alpha - (\alpha' - \frac{1}{L}\nabla f(\alpha'))\right\|_2^2 + \|\alpha\|_1$$

The proposed algorithm:

$$\alpha^{k+1} = prox_{\lambda\Psi}(\beta^k - \frac{1}{L}\nabla f(\beta^k))$$

$$t^{k+1} = \frac{1+\sqrt{1+4t^k}}{2}$$

$$\beta^{k+1} = \alpha^k + \frac{t^k - 1}{t^{k+1}}(\alpha^{k+1} - \alpha^k)$$

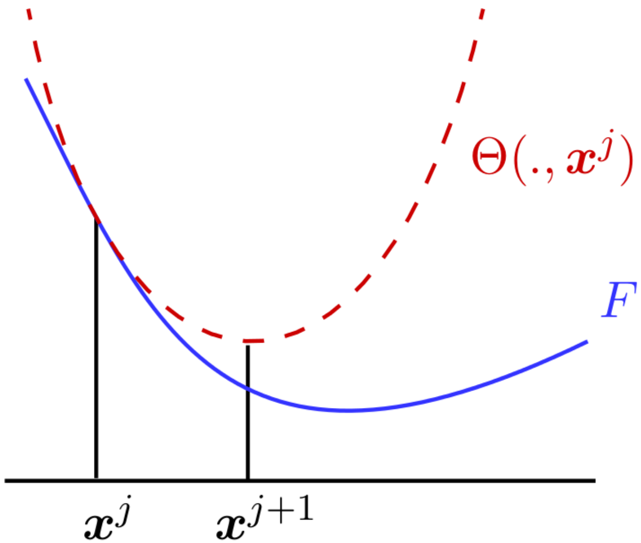
Replace a tricky optimization problem by a simpler:

$$Find\ \hat{x} \in Argmin\ F$$

$$\forall\ x',\ let\ \Theta(.,x')\ a\ tangent\ majorant\ of\ F\ at\ x'$$

$$\forall\ x, \Theta(x,x') \geq F(x')$$

$$\Theta(x',x') = F(x')$$



Build a majorizing surrogate: $F(x) \leqslant \Theta(x, x')$

$$\text{minimize } F(x) = f(Hx - y) + \Psi(x)$$

$$\text{minimize } F(x) = f(Hx - y) + \sum \psi_s(x)$$

Under some assumptions:

The majorant function is:

$$\Theta(x, x') = F(x') + 2\Re\left(\nabla F(x')^H (x - x')\right) + \frac{1}{2}(x - x')^H A(x')(x - x')$$

$$A(x) = \mu H^H H + \Phi^H \, Diag\left(\frac{\dot{\Psi}(|\Phi x|)}{|\Phi x|}\right)\Phi$$

EVALUATION OF THE STEP SIZE AND THE SURROGATE

- How can we make the comparison?

With a proximable differentiable function as the penalization

- Compare the quality of the majorizing function
- Compare the step size

Setting the same stopping criterion as:

- Difference between two consecutive values of the objective lower than a given threshold

IDENTIFY THE CONVENIENT PENALIZATION

- L_2-L_1 approach
  - Comparison of the two algorithms using the same penalization:

$$\Psi(u) = \frac{|u|}{\delta} - \log\left(\frac{|u|}{\delta} + 1\right)$$

$$\nabla \Psi(u) = \frac{1}{\delta} \frac{u}{|u| + \delta}$$

$$prox_{\lambda \Psi}(u) = 0.5 * sign(u)\left(|u| - \frac{\lambda}{\delta} - \delta + \sqrt{\left(|u| - \frac{\lambda}{\delta} - \delta\right)^2 + 4\,\delta\,|u|}\right)$$

QUANTITATIVE COMPARISON BETWEEN FISTA & 3MG

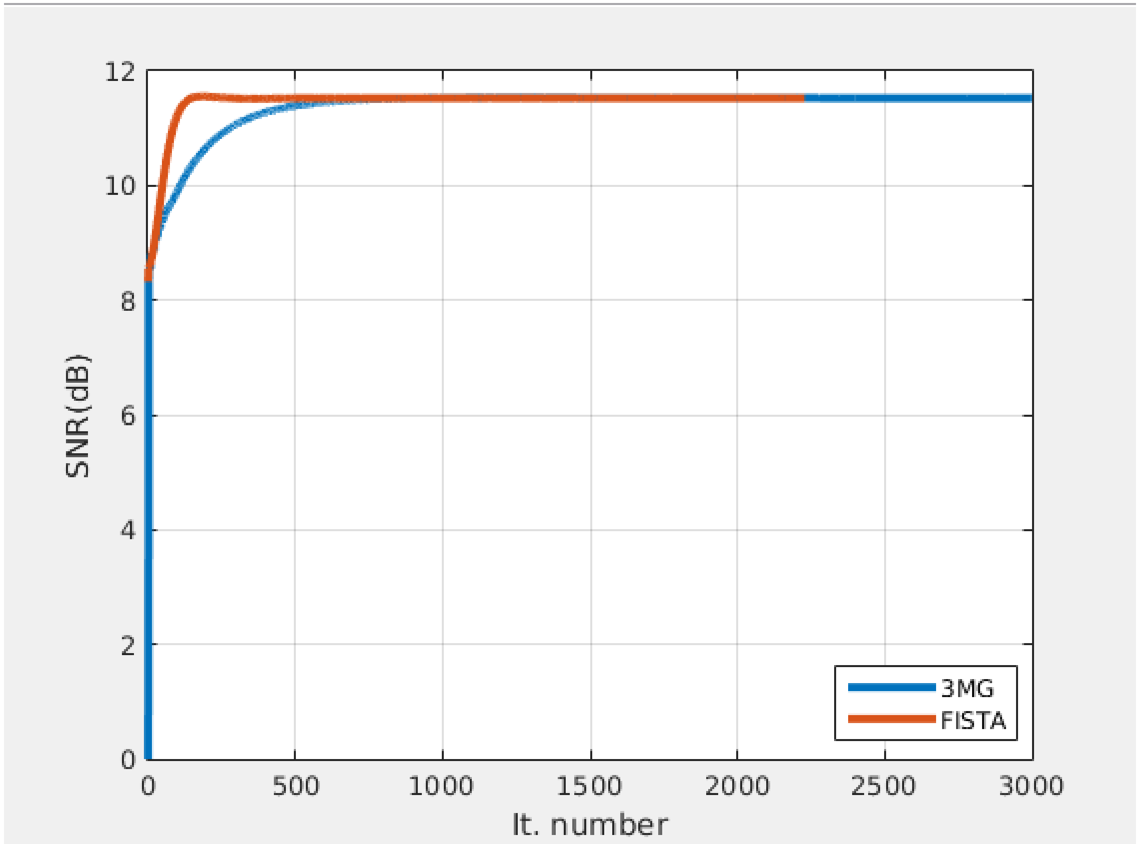
- Using the same inputs and the same regularization parameters:

	3MG	FISTA
First crit. value	8.1873+07	8.1873+07
Last crit. value	6.9592e+04	6.9503e+04
Final SNR	11.5258	11.5256
Time to convergence (s)	231.1135	201.0264
Number of iteration	3000	2228

EVOLUTION OF THE SNR ALONG ITERATIONS

- For the same input values (same noise)

Evolution of the SNR



EVOLUTION OF THE SNR OVER TIME

- For the same input values (same noise)

Evolution of the SNR as a function of time

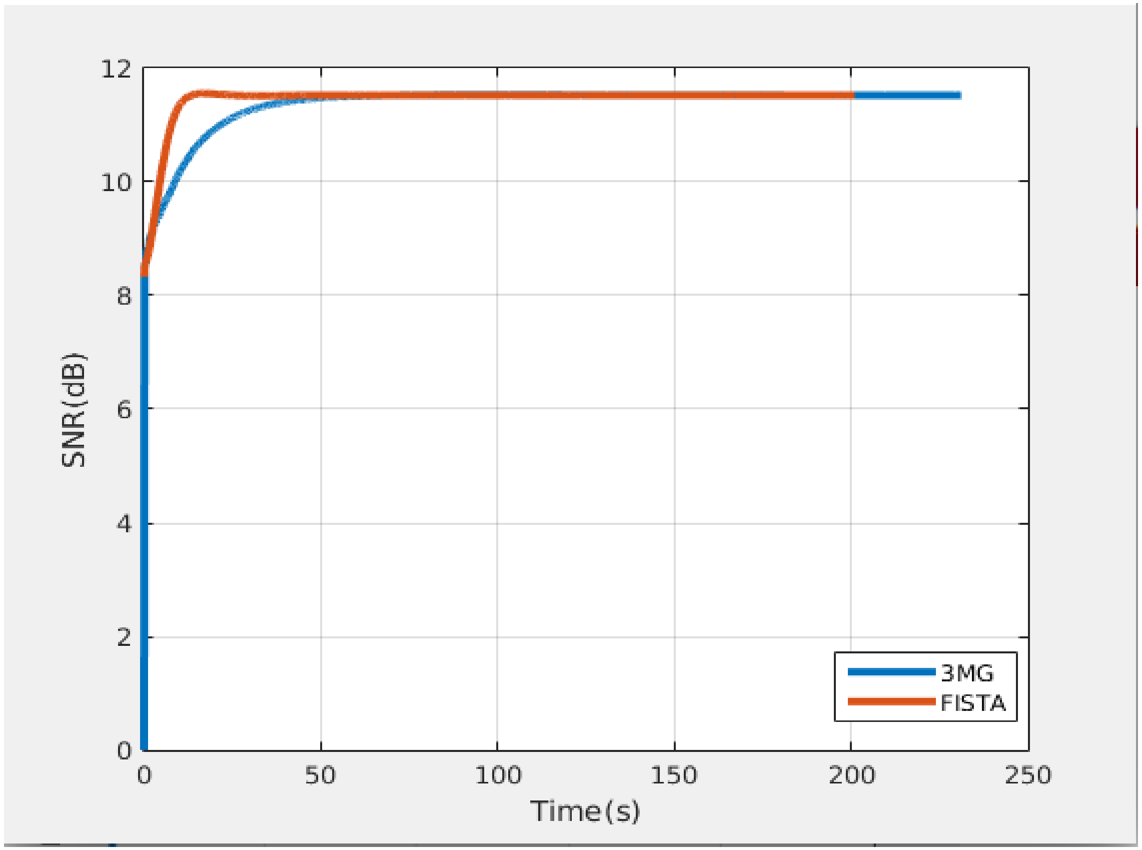


IMAGE RECONSTRUCTION (FISTA)

- For the same input noisy data + same parameters

Reference

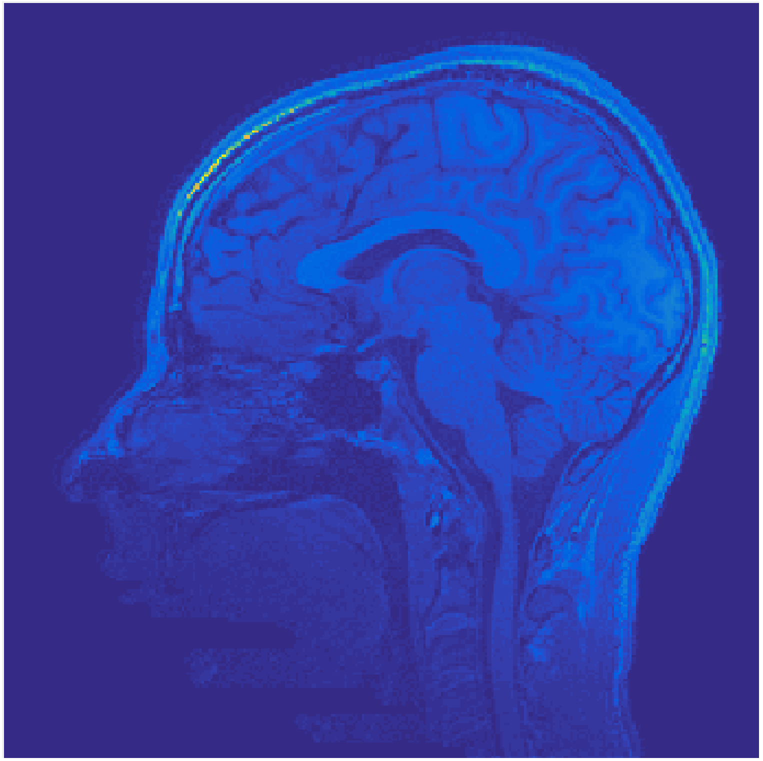


Image solution

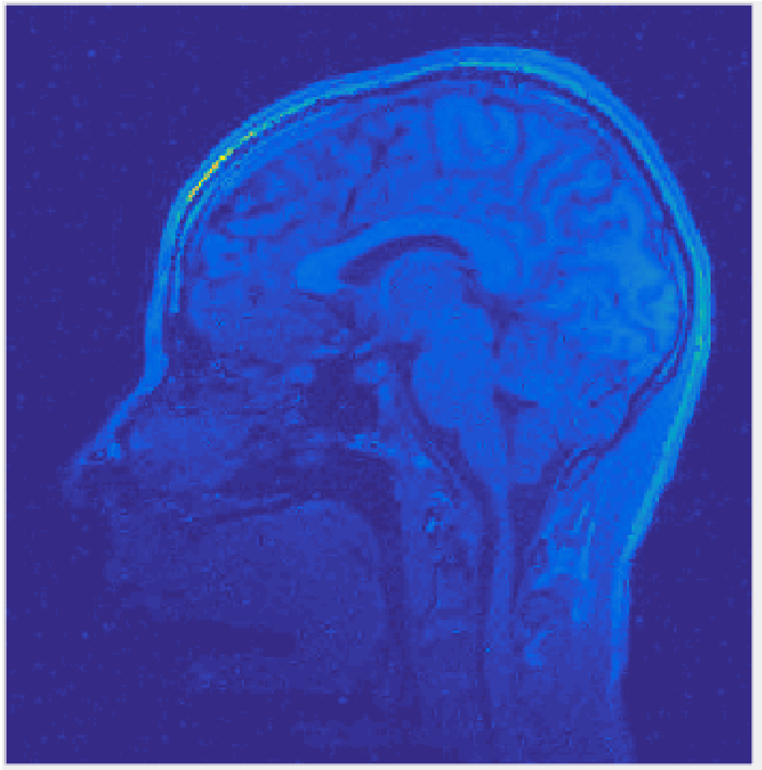


IMAGE RECONSTRUCTION (3MG)

For the same input noisy data + same parameters

Reference

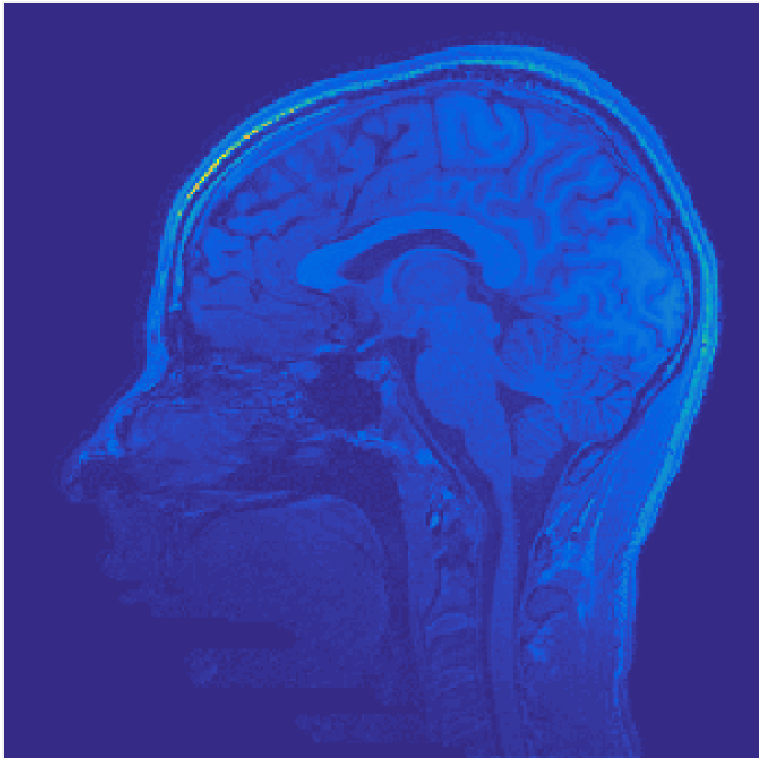
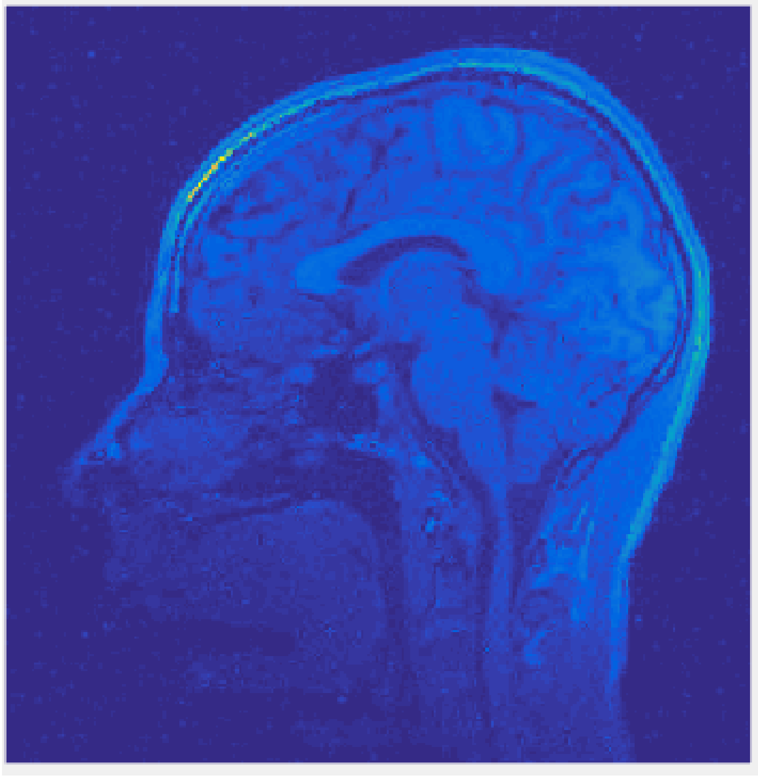


Image solution



Both FISTA and 3MG converge to the same solution

EVALUATION OF THE STEP SIZE AND THE SURROGATE

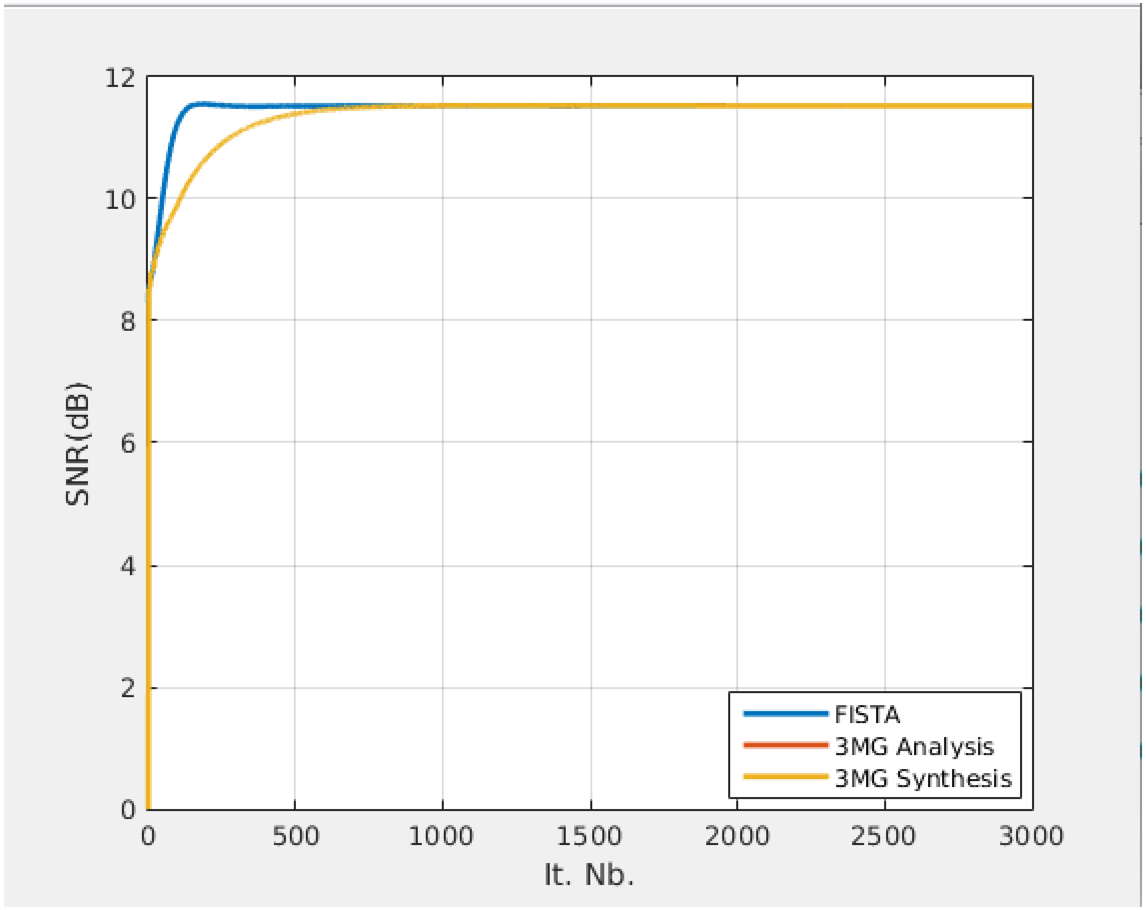
How can we explain the transition mode of the 3MG :

- Analysis vs Synthesis formulation
- Impact of the memory gradient
- Diminution of the computation time

IMPACT OF THE SYNTHESIS FORMULATION

- For the same input values, synthesis formulation for FISTA and 3MG

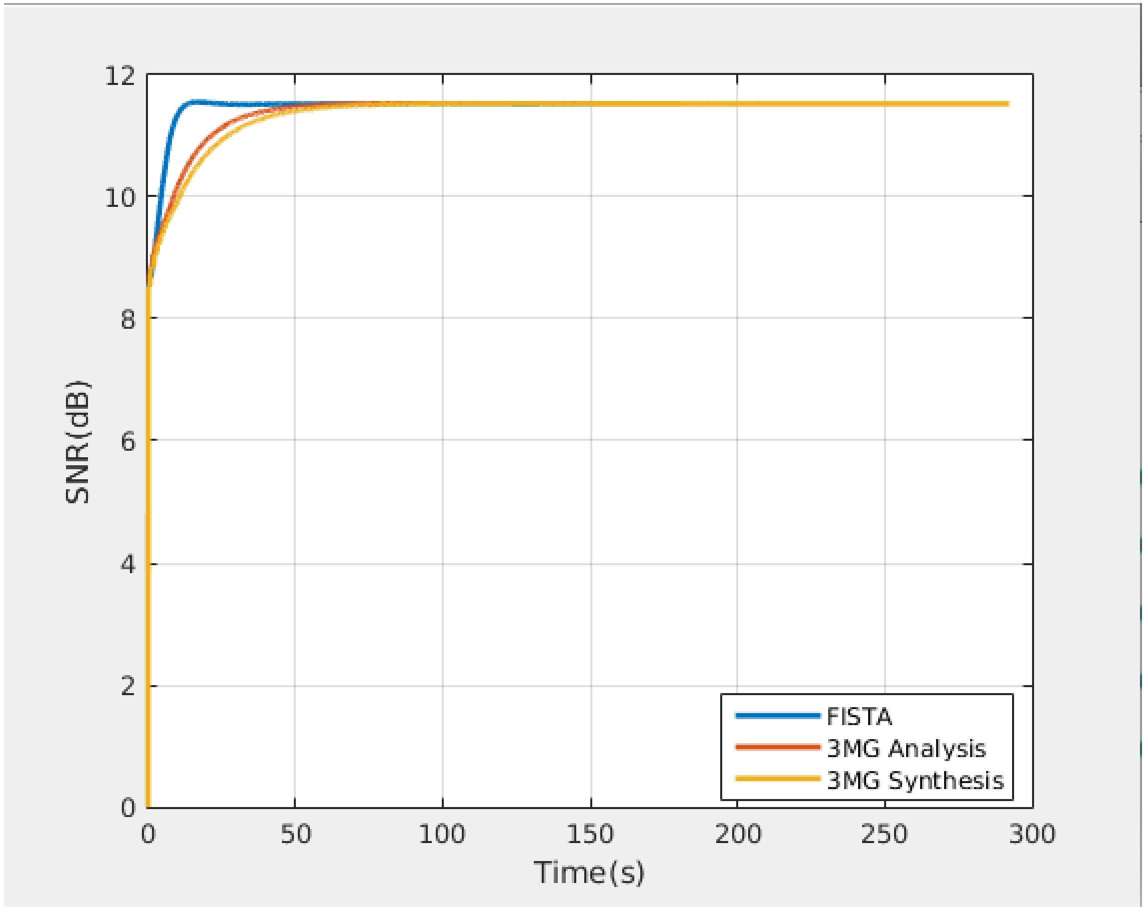
Evolution of the SNR



IMPACT OF THE SYNTHESIS FORMULATION

- For the same input values, synthesis formulation for FISTA and 3MG

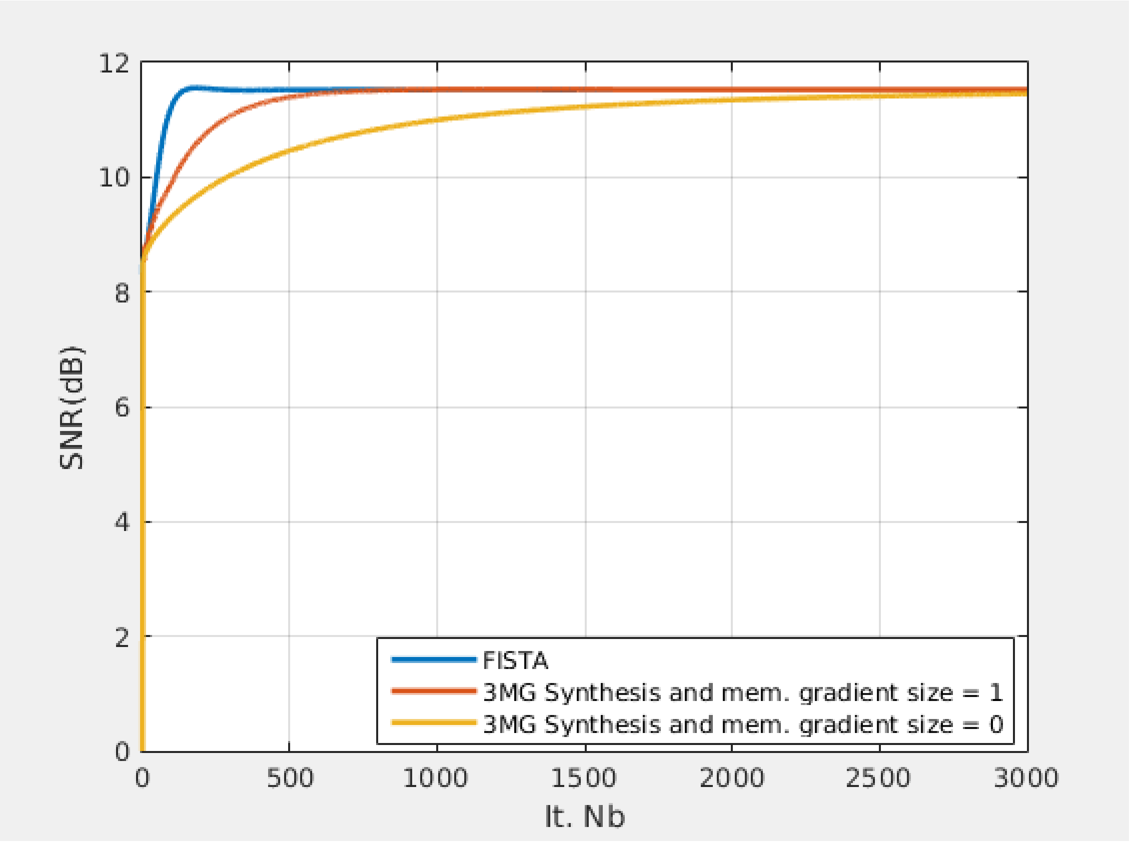
Evolution of the SNR (time)



IMPACT OF THE MEMORY GRADIENT

- Without memory (synthesis formultaion)

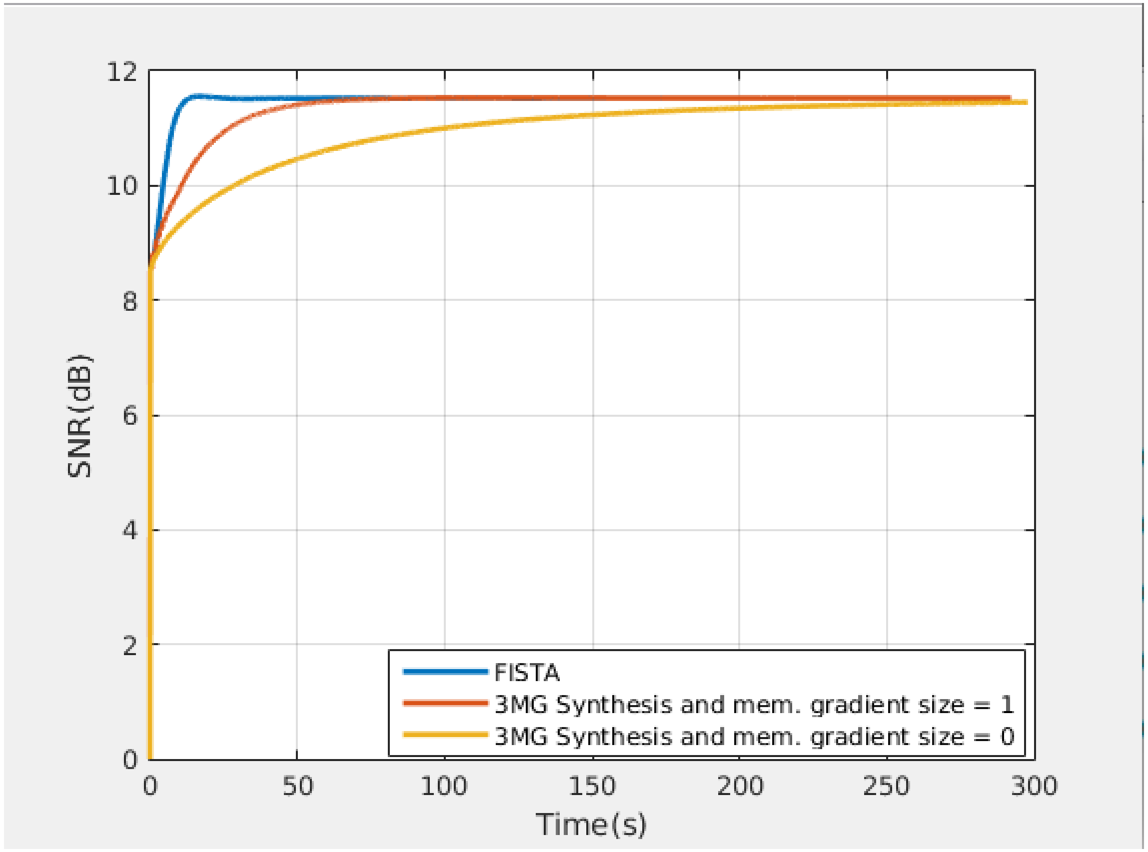
Evolution of the SNR



IMPACT OF THE MEMORY GRADIENT

- Without memory (synthesis formultaion)

Evolution of the SNR (Time)



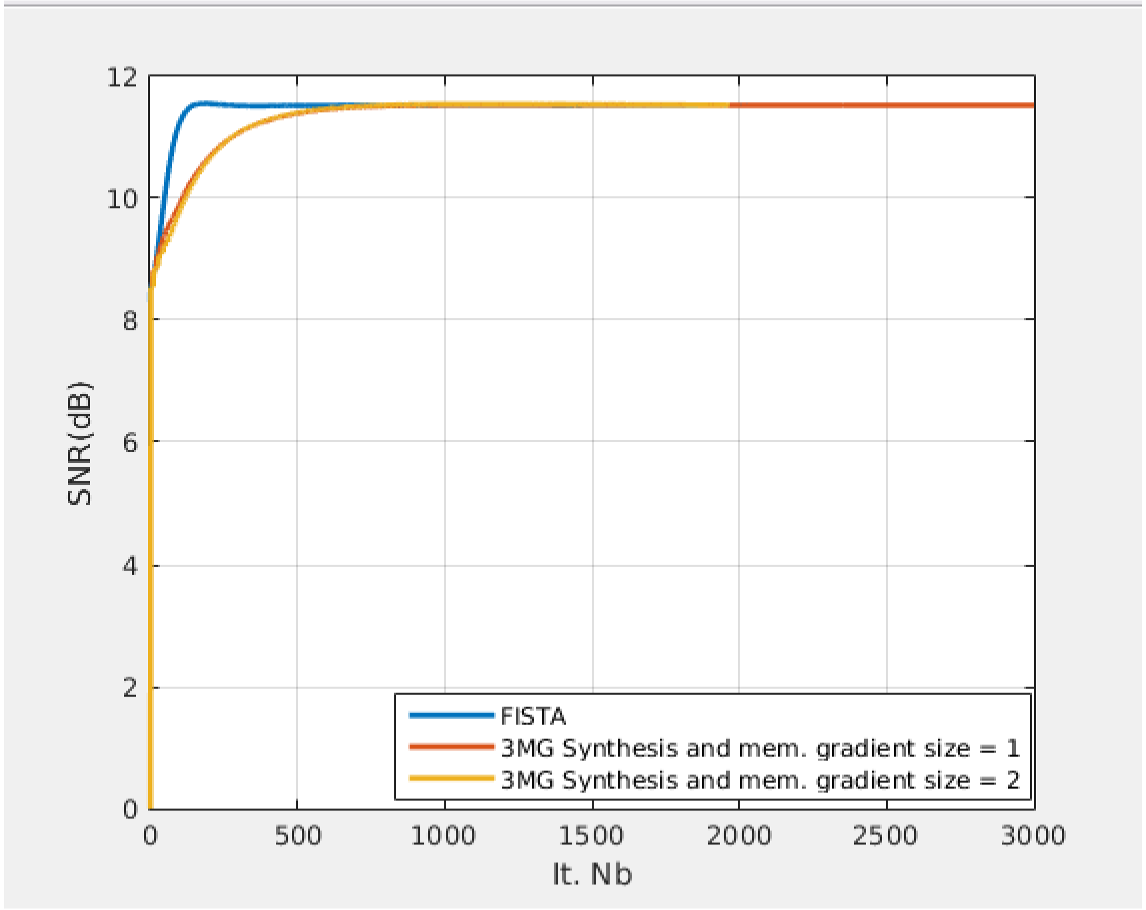
IMPACT OF THE MEMORY GRADIENT

Increase the memory impact?

IMPACT OF THE MEMORY GRADIENT

- Memory size = 2 (synthesis formulation)

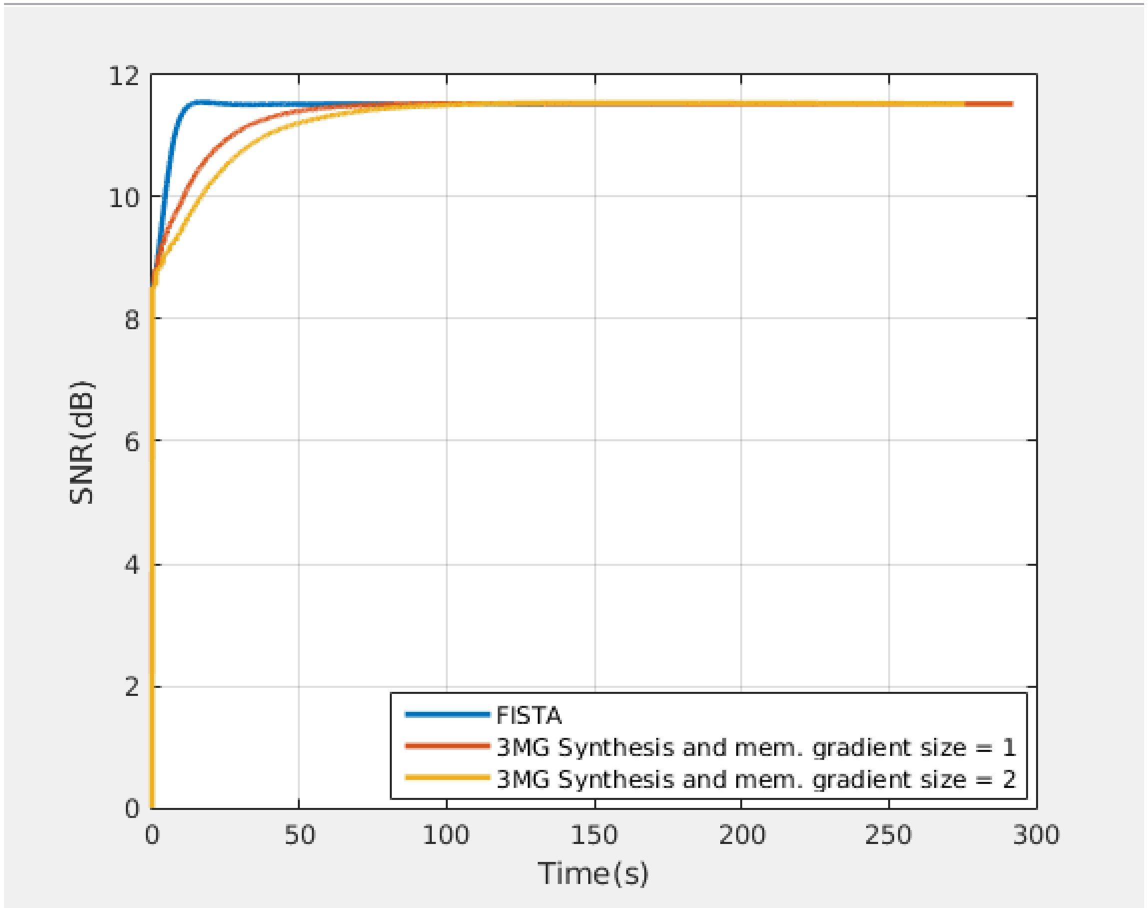
Evolution of the SNR



IMPACT OF THE MEMORY GRADIENT

- Memory size = 2 (synthesis formulation)

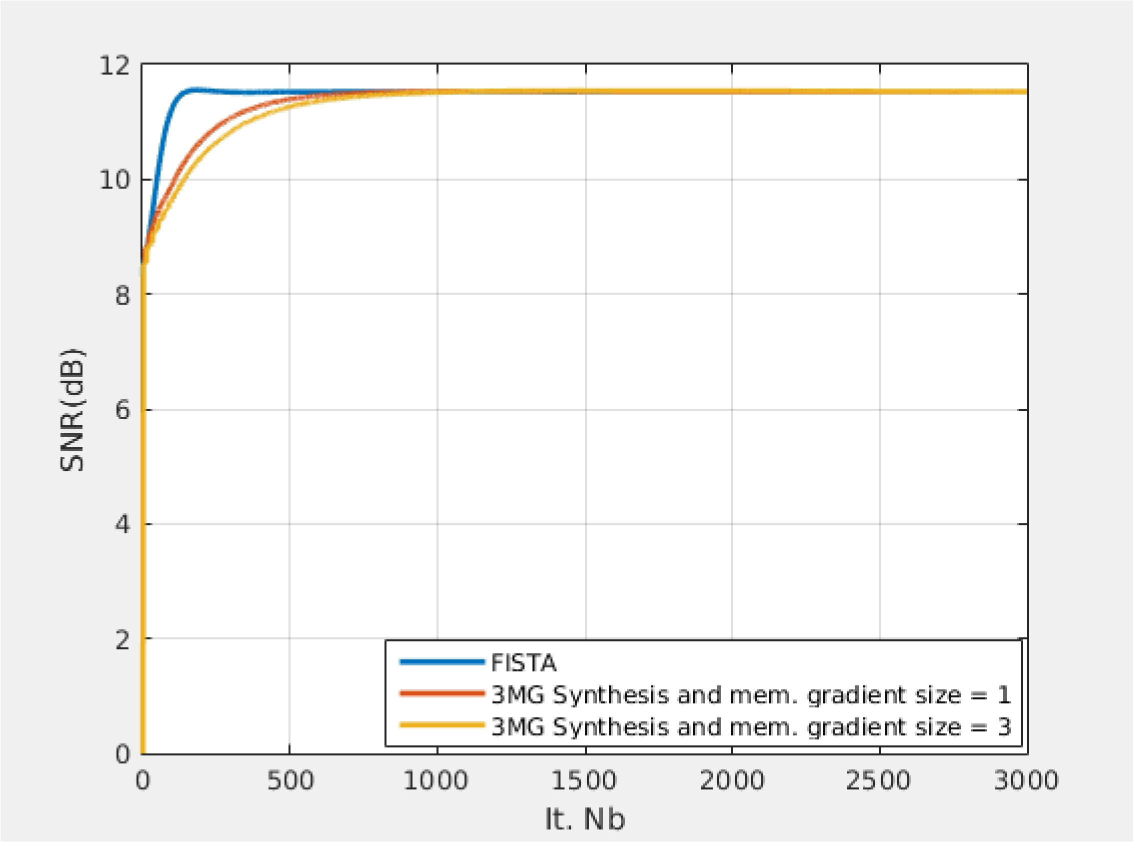
Evolution of the SNR (Time)



IMPACT OF THE MEMORY GRADIENT

- Memory size = 3 (synthesis formulation)

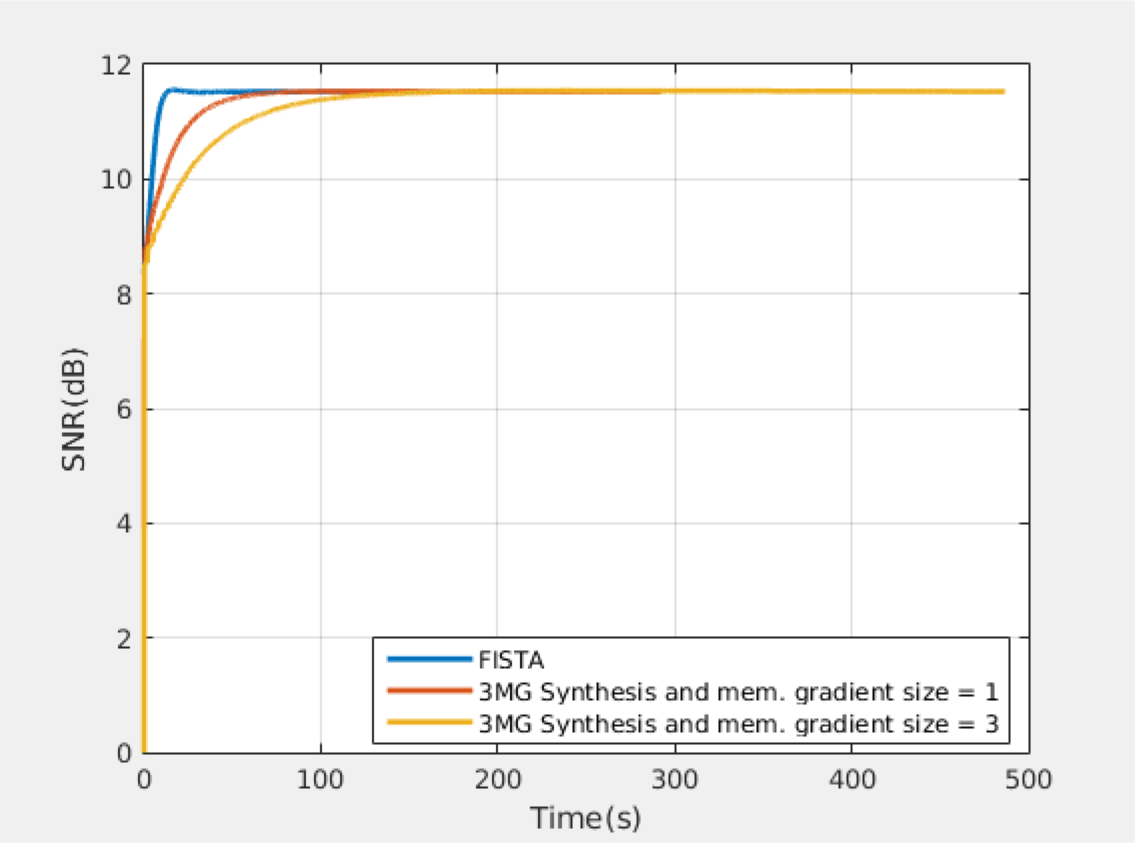
Evolution of the SNR



IMPACT OF THE MEMORY GRADIENT

- Memory size = 3 (synthesis formulation)

Evolution of the SNR (Time)



SPEED UP THE RECONSTRUCTION

Reduce majorant complexity?

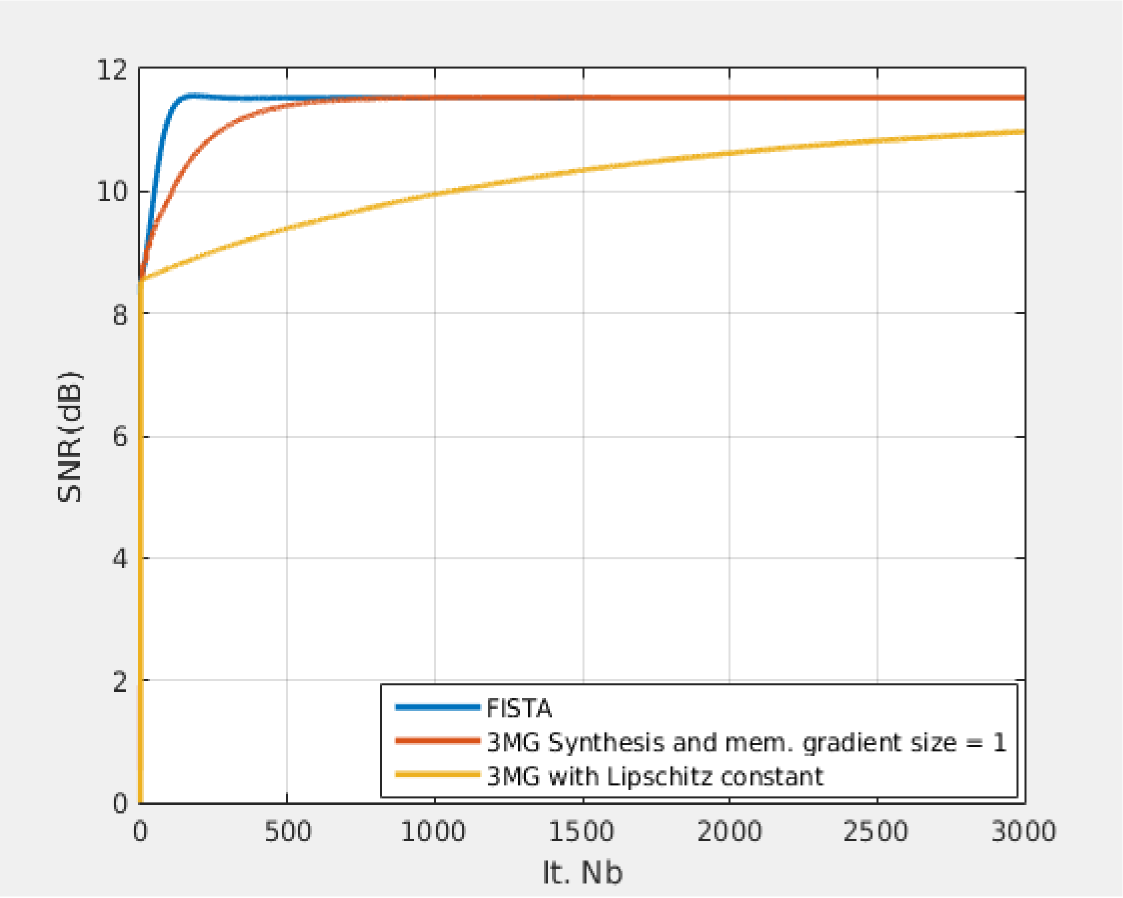
Majorize the surrogate by the Lipschitz constant of the quadratic term

$$\Theta(x, x') = F(x') + 2\Re\left(\nabla F(x')^H (x - x')\right) + \frac{1}{2}(x - x')^H A(x')(x - x')$$

$$A(x) = \mu (Hx)^H (Hx) + \Phi^H Diag\left(\frac{\dot{\Psi}(|\Phi x|)}{|\Phi x|}\right)\Phi$$

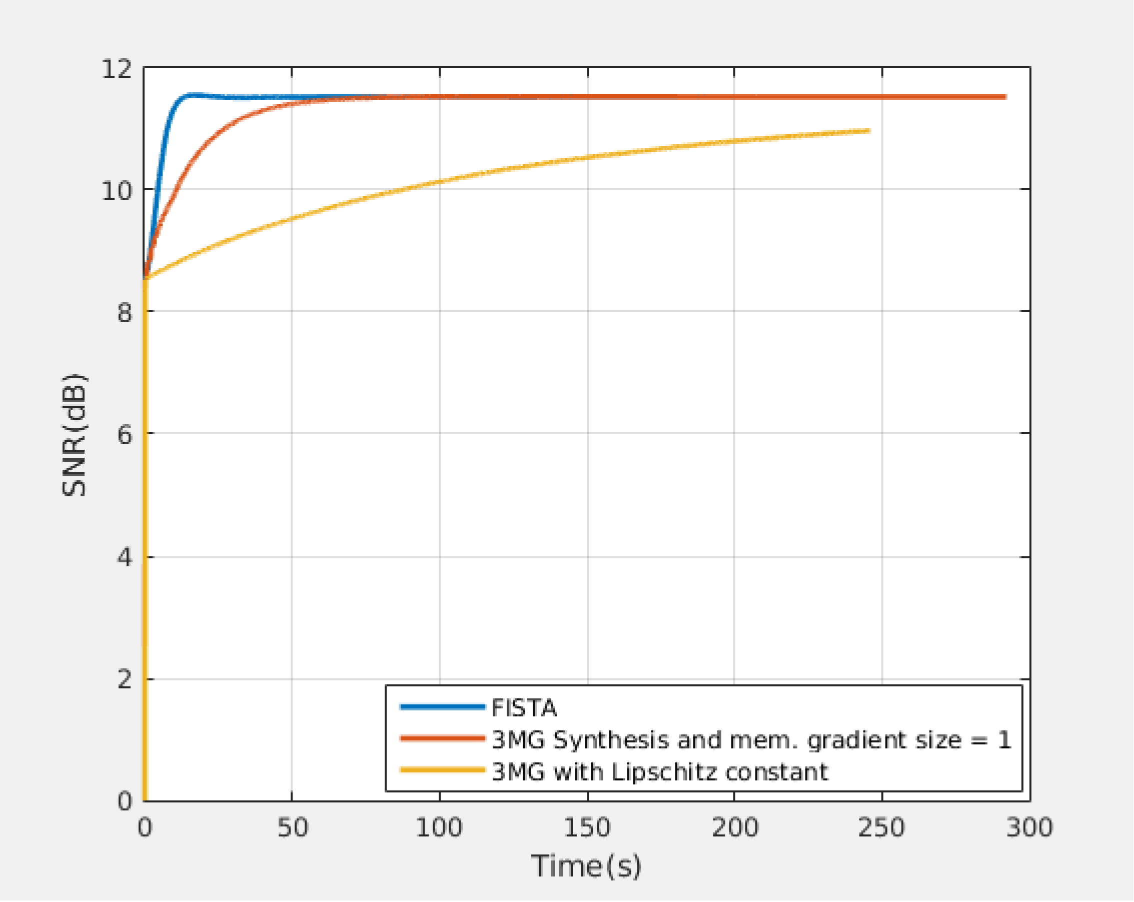
- Using the Lipschitz constant (synthesis formulation)

Evolution of the SNR



- Using the Lipschitz constant (synthesis formulation)

Evolution of the SNR (Time)

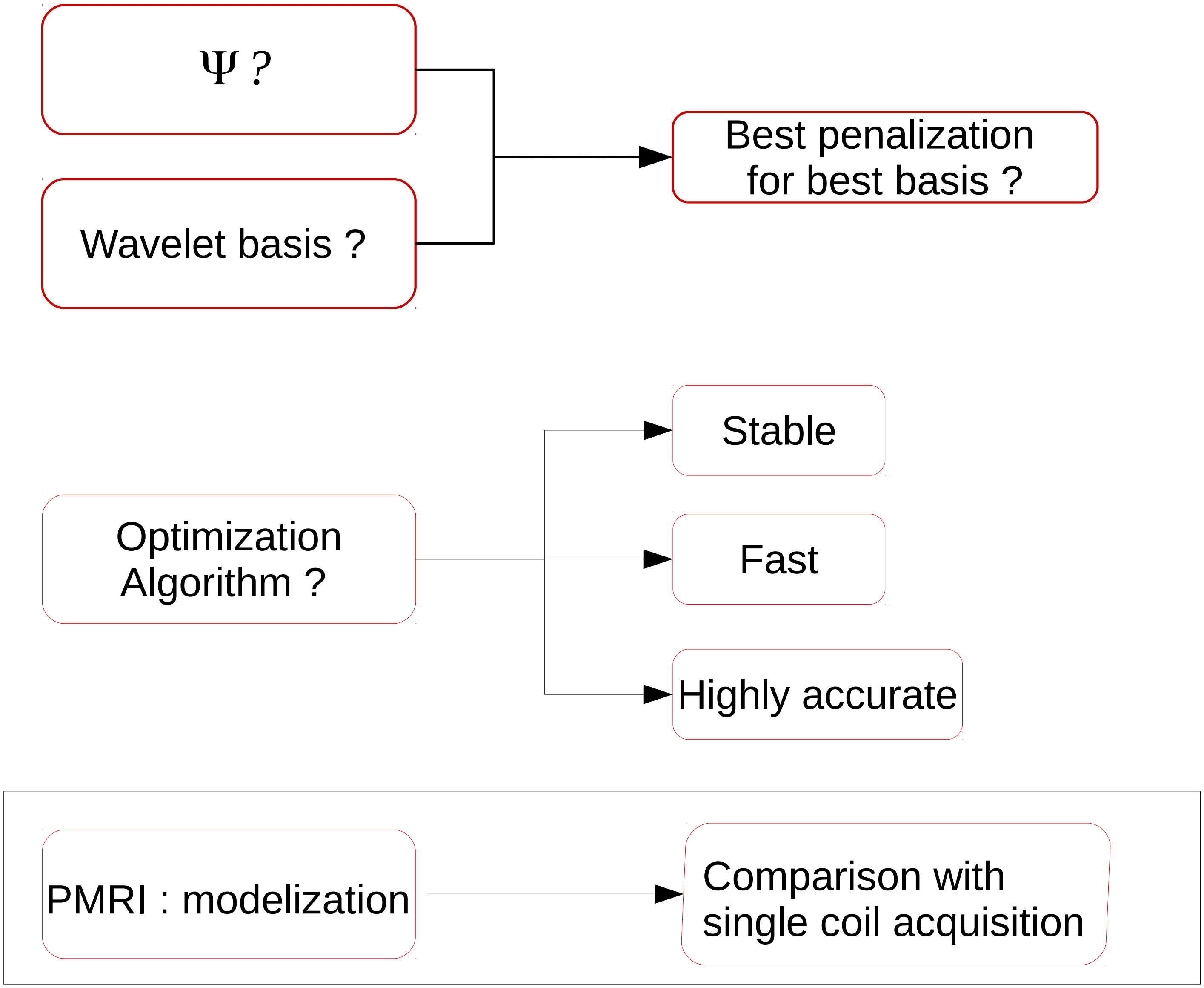


CONCLUSION

We should probably investigate the proximal algorithm :

Current work: Adaptation of a primal-dual algorithm Condat-vu

RECONSTRUCTION PROBLEM OVERVIEW

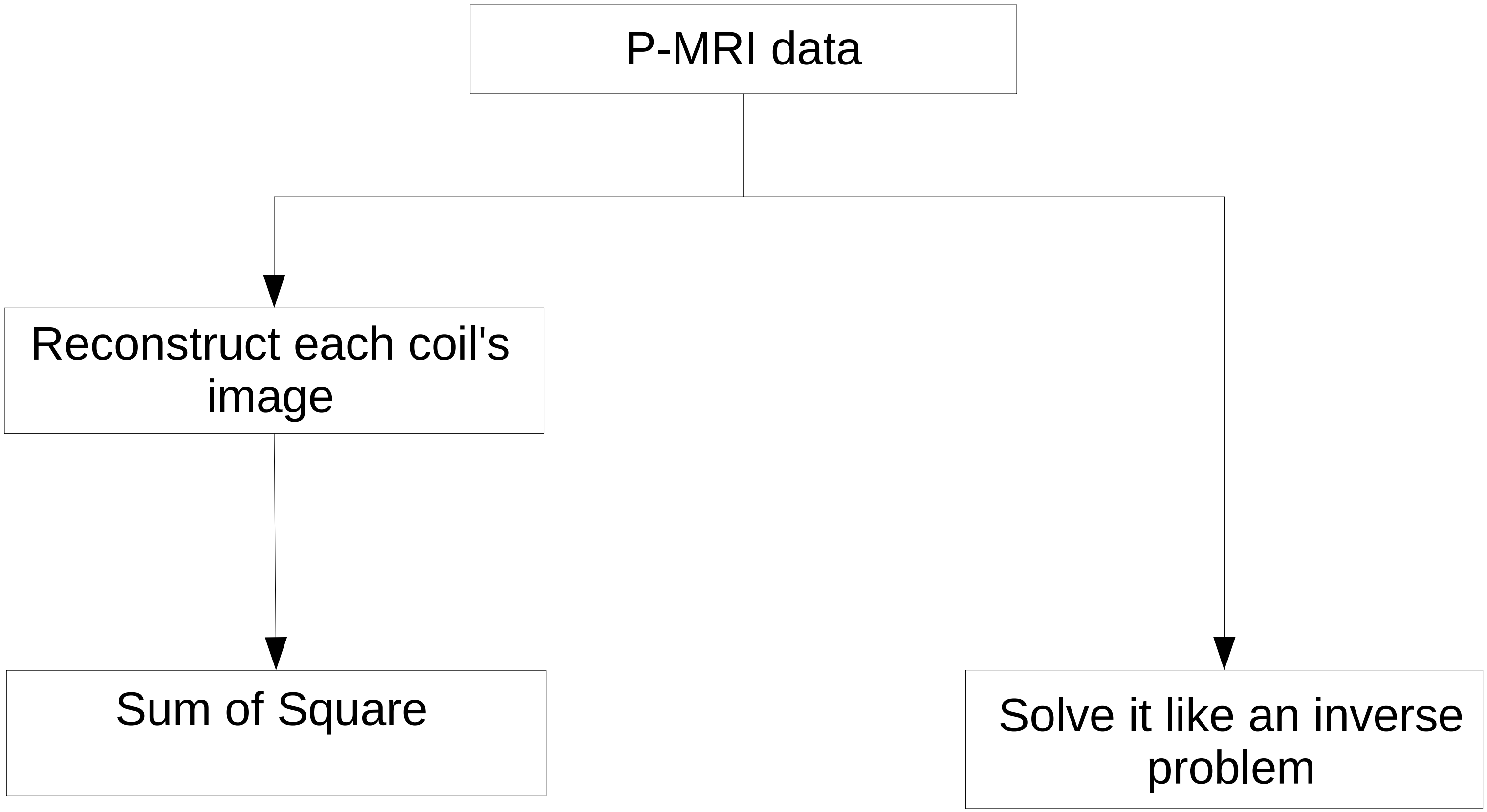


PMRI: PROBLEM

- Advantage: Improve the signal-to-noise ratio (SNR)
- Drawbacks:
  - Increased computation time at the reconstruction stage

PMRI: PROBLEM

- Regarding the reconstruction :



PMRI: PROBLEM

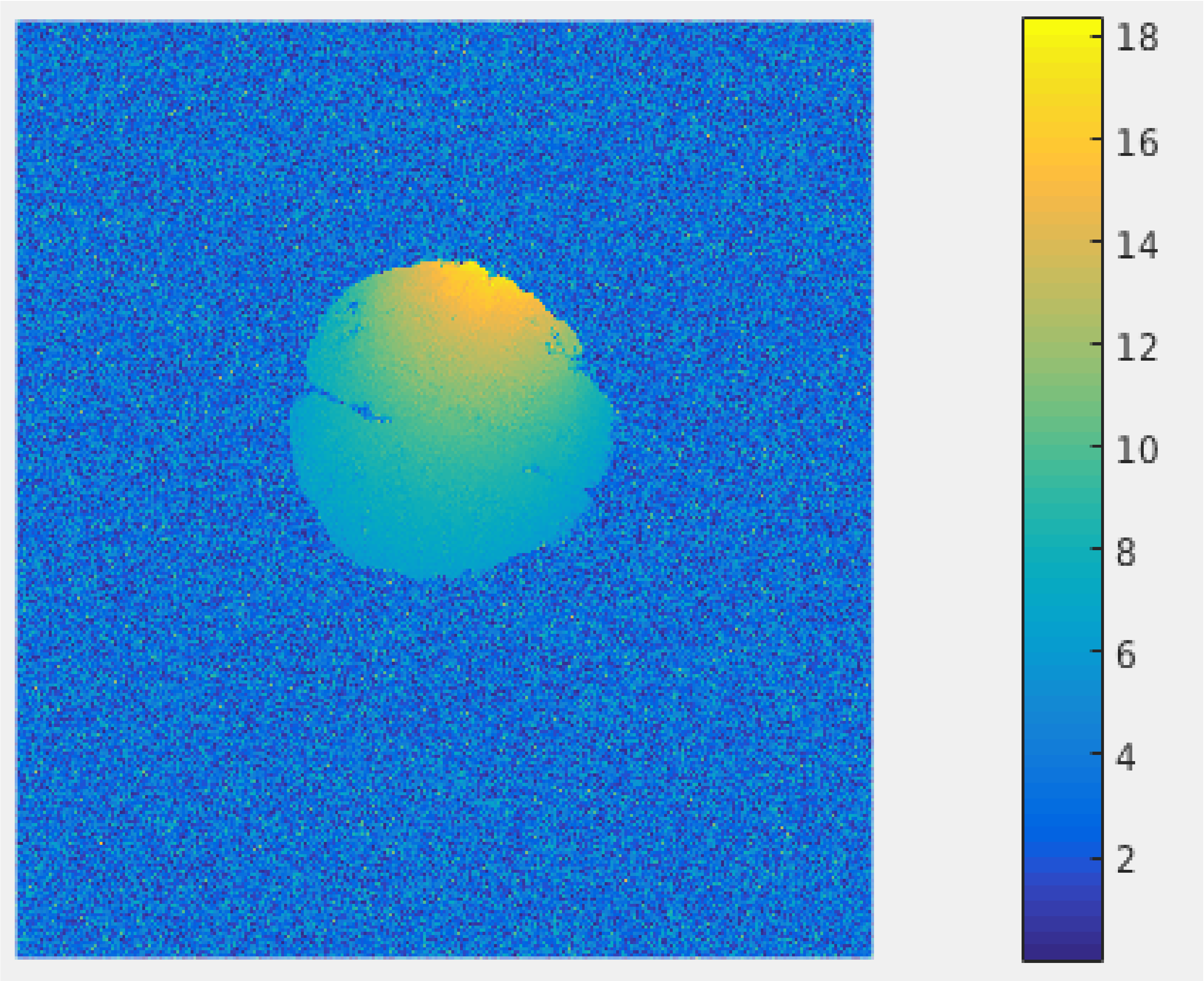
- Advantage: use the information given by each coil
- Drawbacks:
  - Require the knowledge of sensitivity maps

$$\hat{\alpha} = argmin \sum_{l=1}^{L} \|F^* S_l \Phi \alpha - y_l\|_2^2 + \lambda \Psi(\alpha)$$

As a first experiment:

The sensitivity matrix will be given by a reference

PMRI: PROBLEM

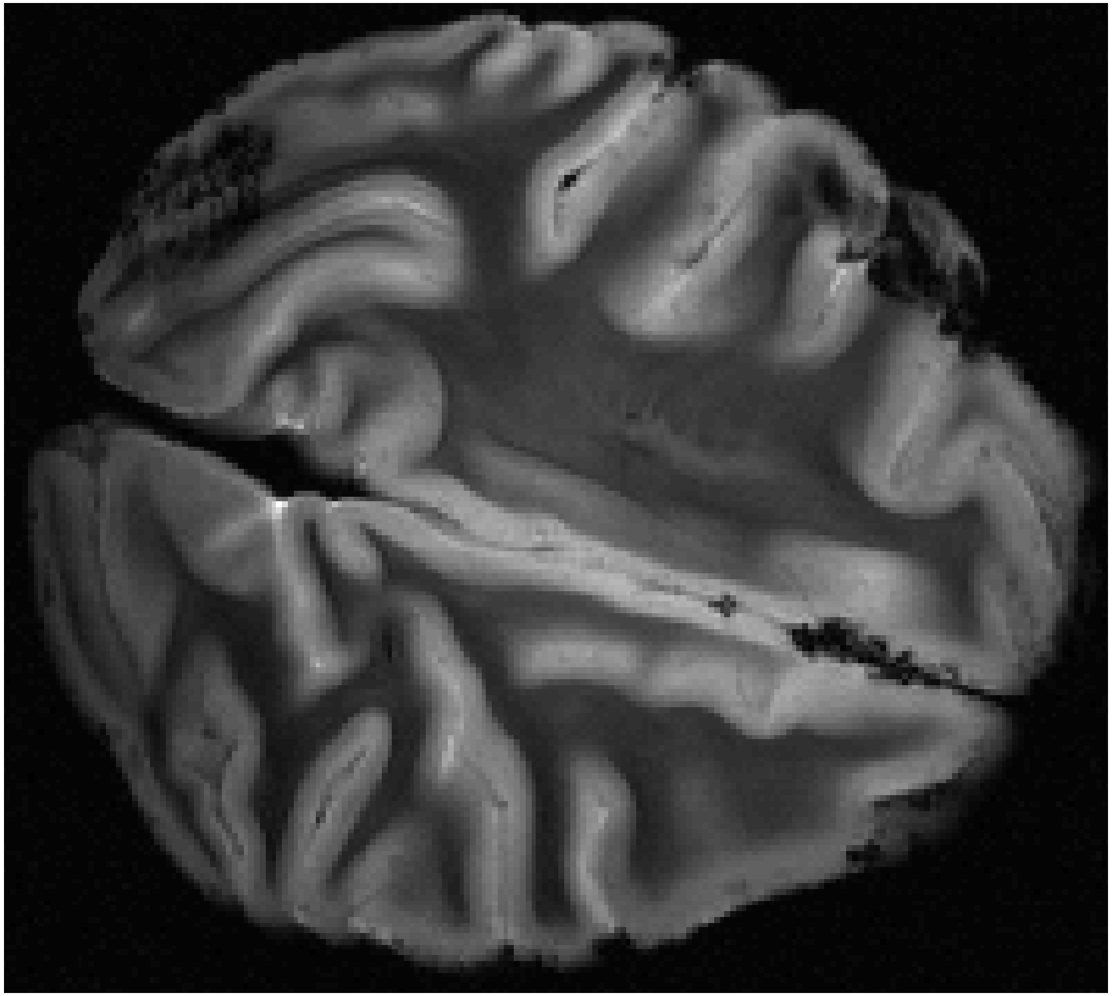


PMRI: PROBLEM

SNR = 43,45

Reference image (SOS)

PMRI: PROBLEM

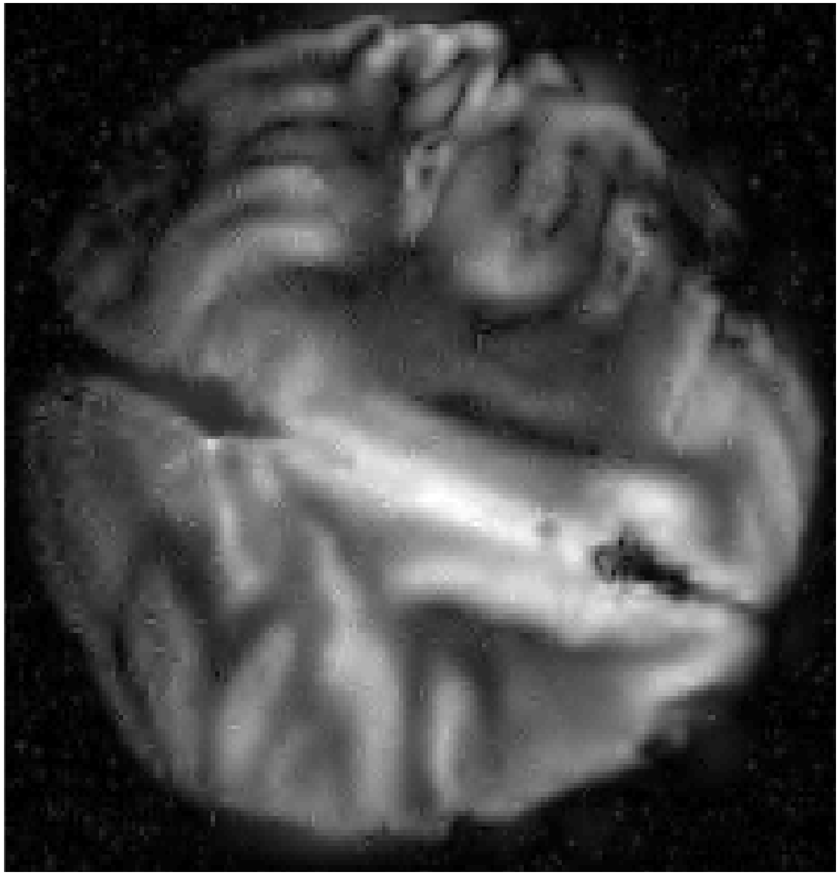


Full K-space

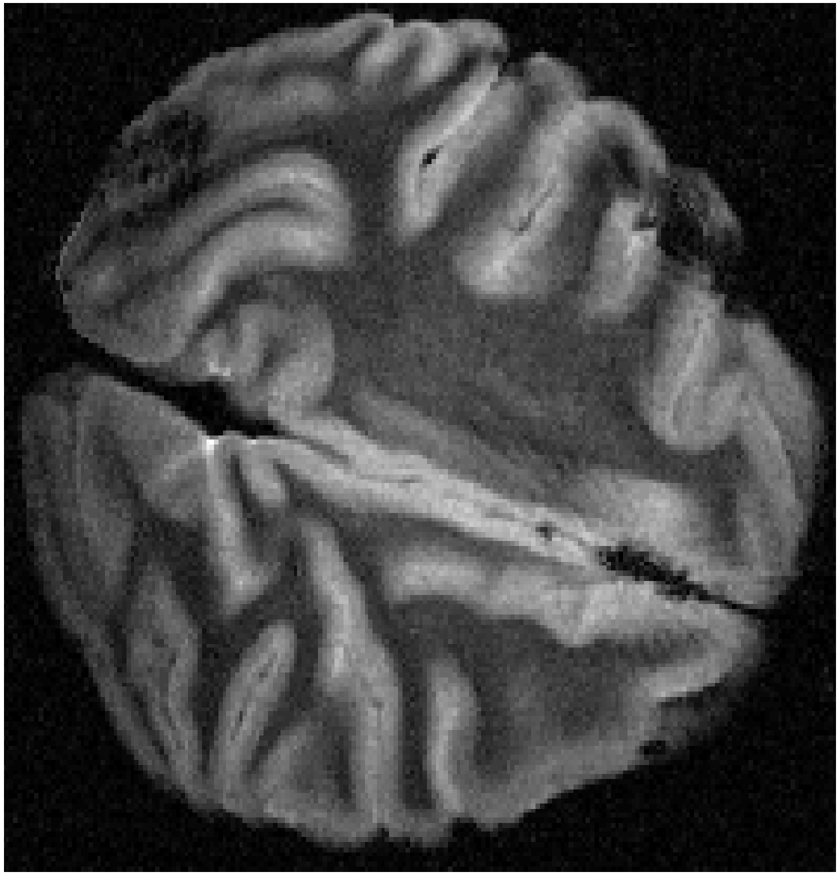
Sparkling
R = 8
N = 512

Reference image (SOS)

PMRI: PROBLEM

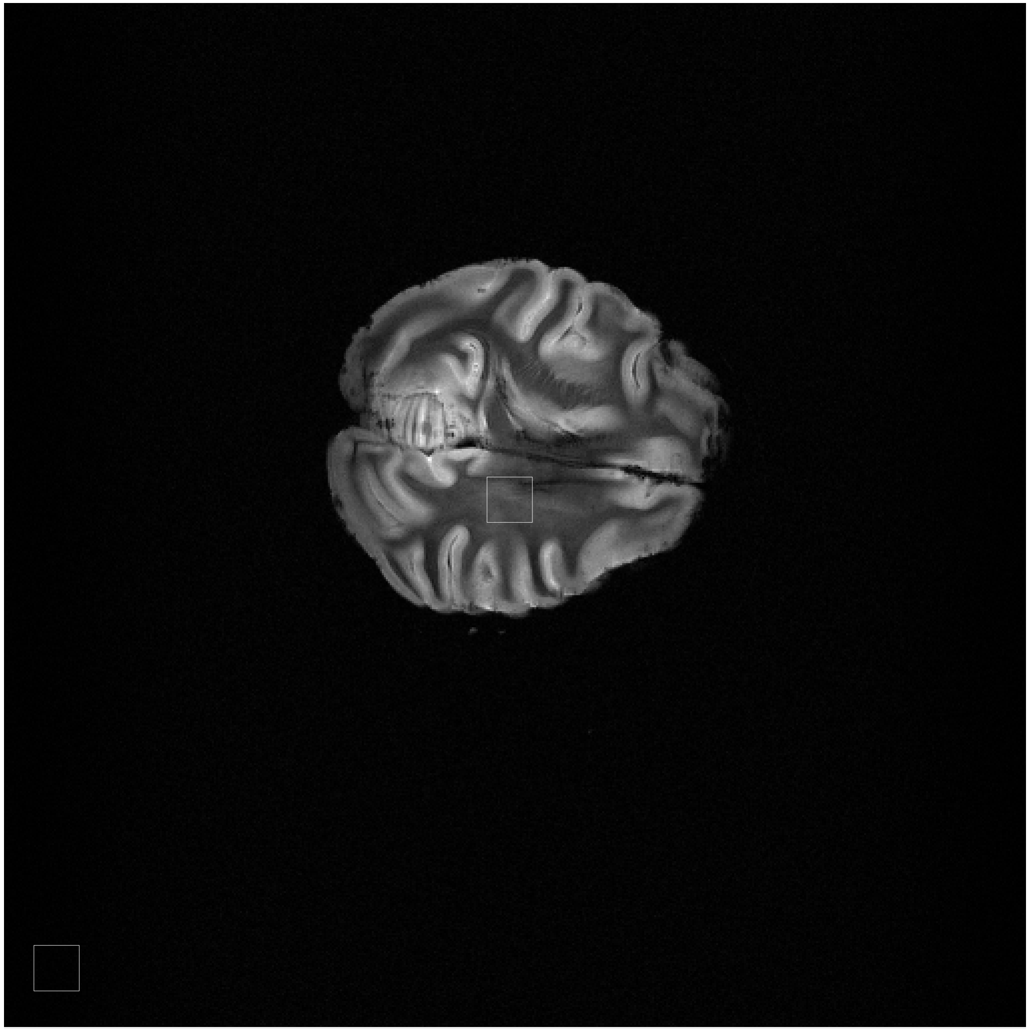


SSIM = 0,67



SSIM = 0,7088

PMRI: COMPARAISON WITH SINGLE COIL

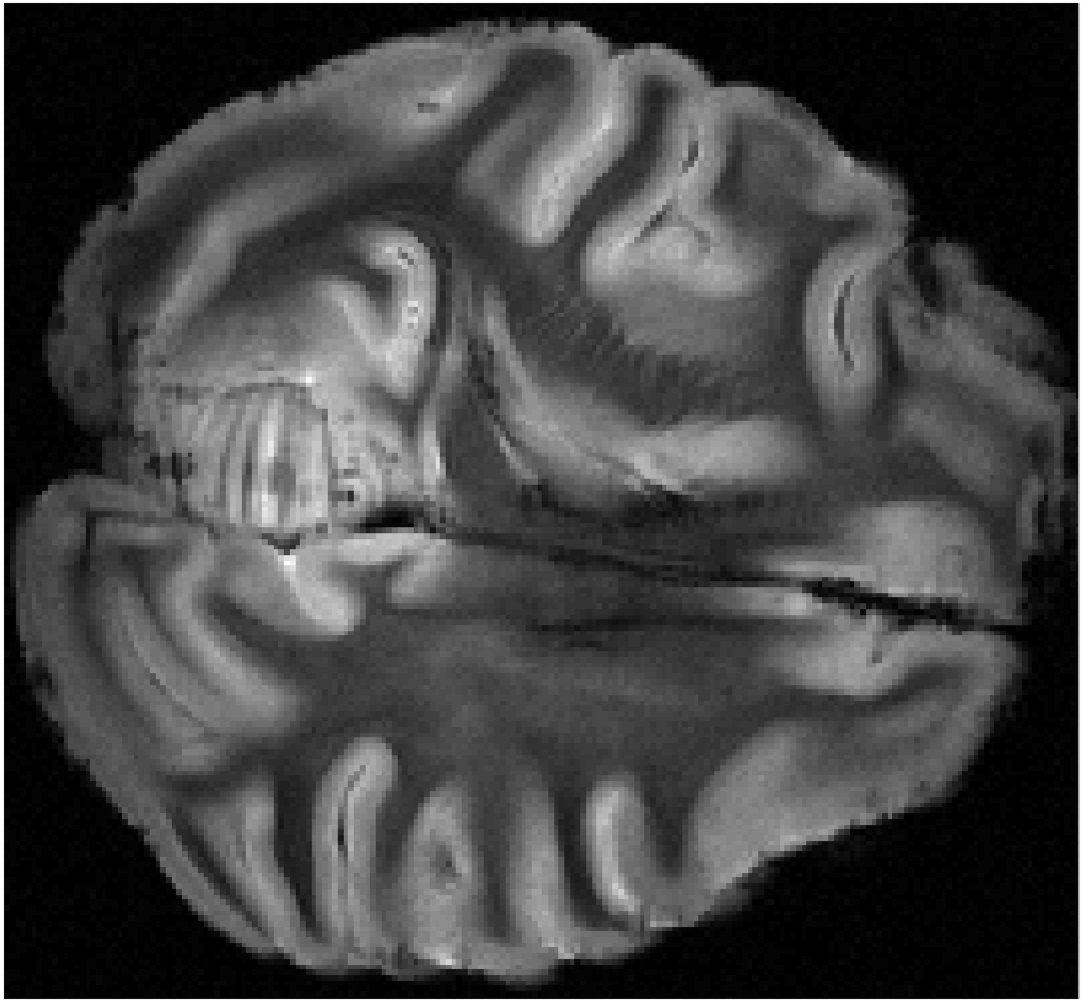


Reference

Full K-space

SNR = 35,4

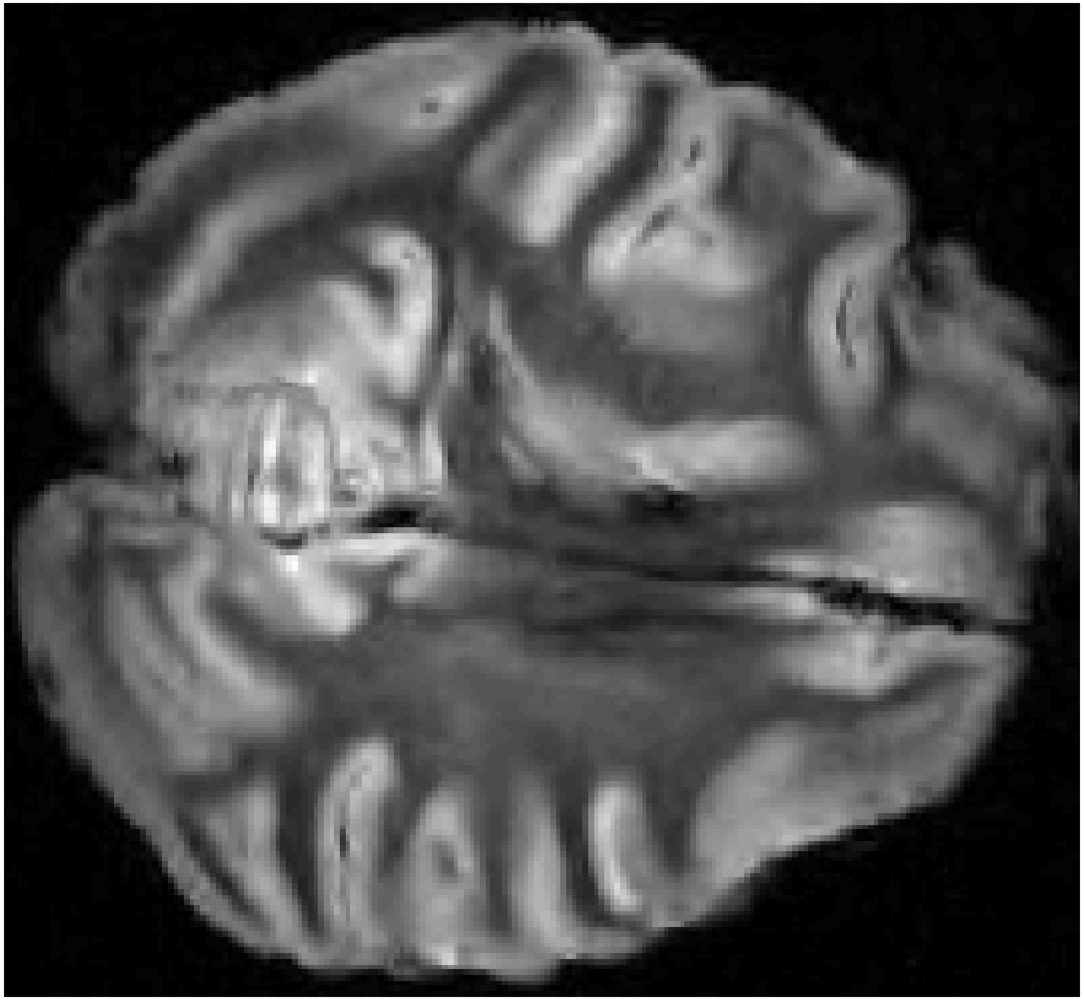
PMRI: COMPARAISON WITH SINGLE COIL



Full K-space

Reference

PMRI: COMPARAISON WITH SINGLE COIL



Reconstruction

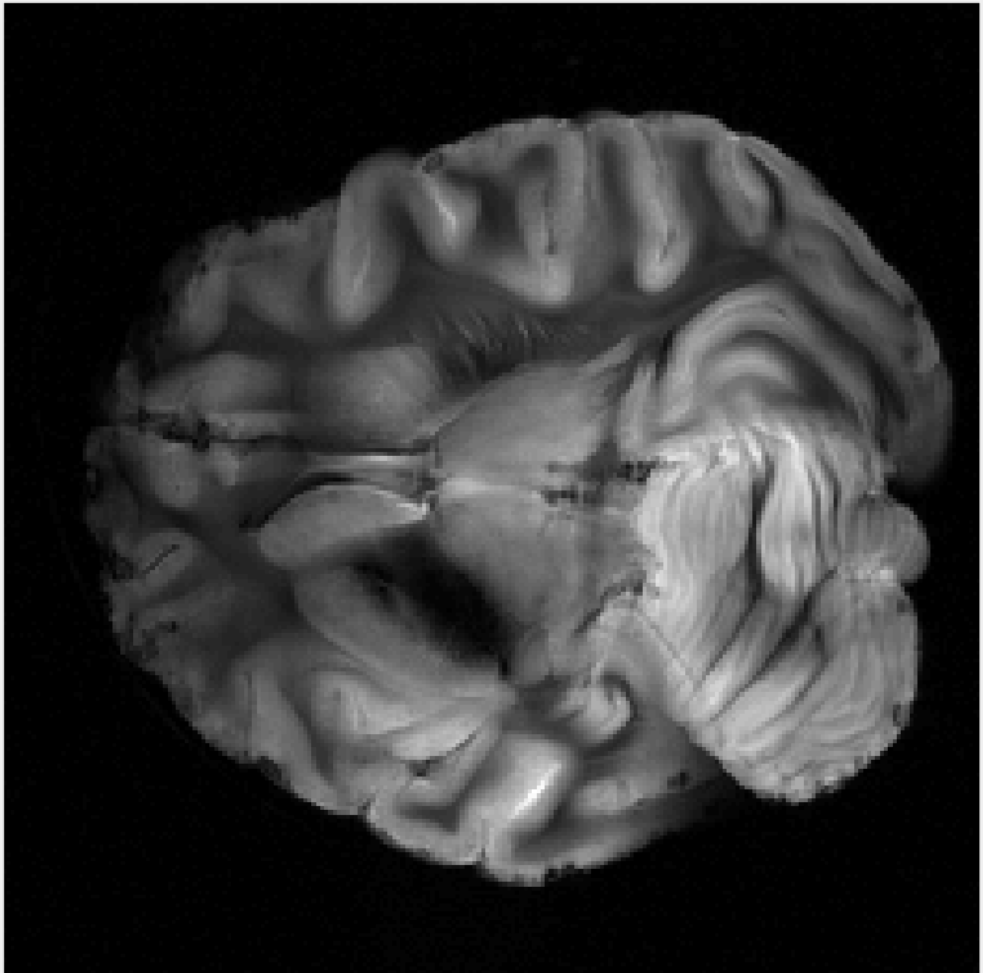
Sparkling
R = 8
N = 512

SSIM = 0,82

PMRI : PROBLEM

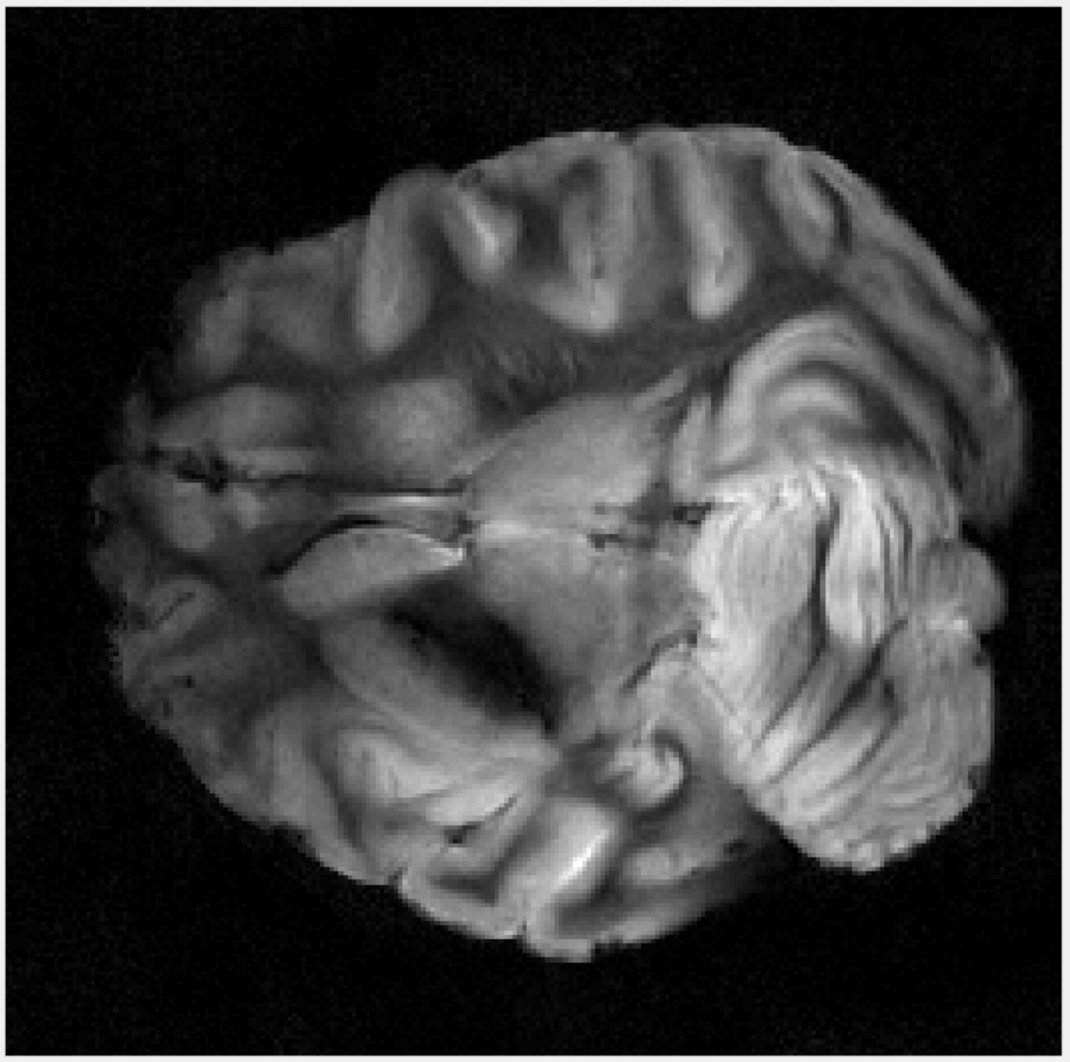
For real acquired data on the 7 Tesla and 32 receiver coils

- Reference image :
  - Full k-space acquisiti
  - Sum of squares
  - Resolution 512x512



PMRI : RESULTS

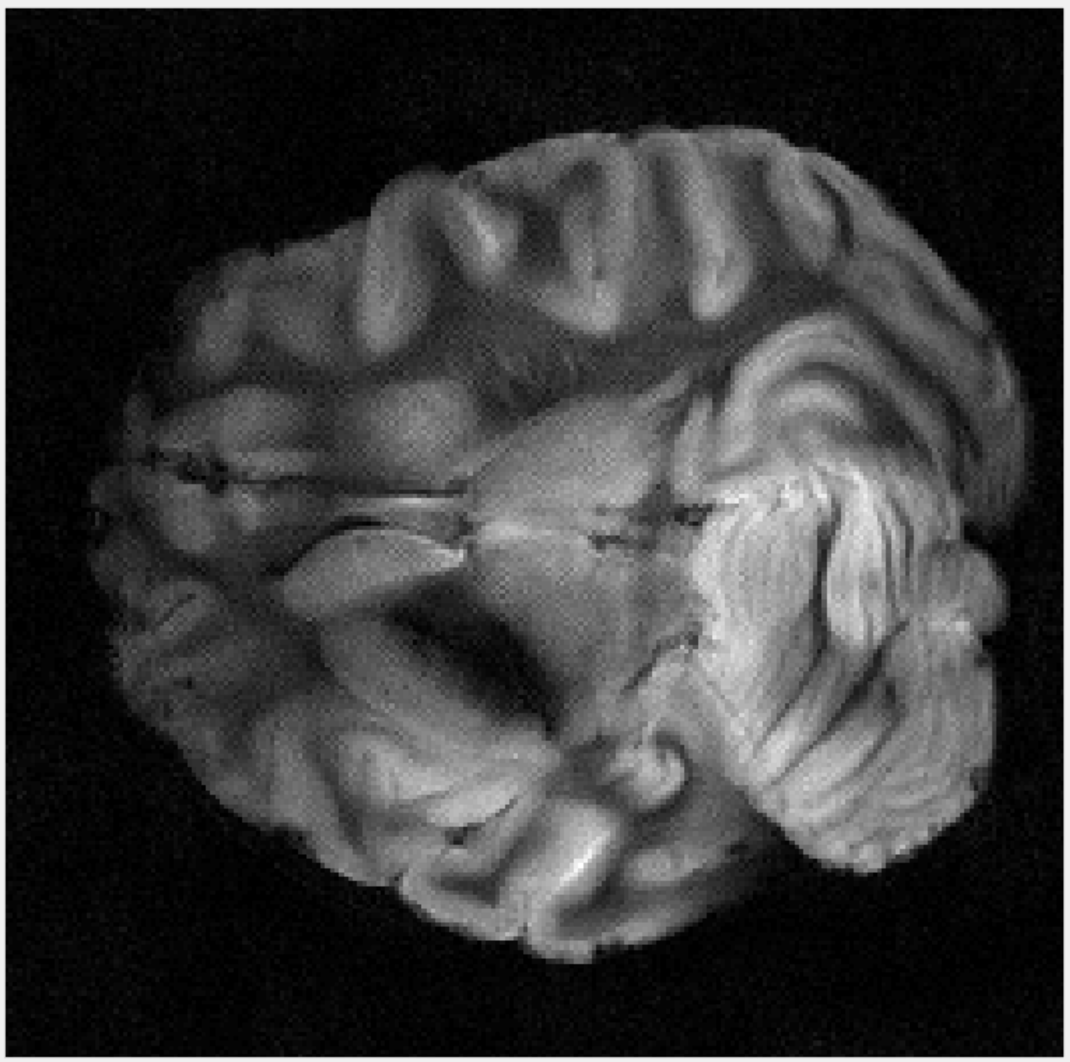
- Reconstruction with FISTA L1



SSIM = 0,74

PMRI : RESULTS

- Reconstruction with 3MG l2-l1



SSIM = 0,68

NEXT STEP

Wavelet basis :

- Test with new basis (undecimated 79 bi-orthogonal basis)

Optimization algorithm :

- Implementation of the Condat-Vu

P-MRI :

- Sensitivity matrix generation:
  - Auto-calibration methods
  - Acquisition of a full K-space low resolution image

CONCLUSION

Thank you for your attention